Title Page (version 0.6)

Classical Electromagnetism: Charge, Current, Energy, and Momentum

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Classical Electromagnetism: Charge, Current, Energy, and Momentum (v. 0.6)

The electrical attractions and repulsion between bodies of measurable dimensions are, of all electrical manifestations of this force, the first so-called electrical phenomena noted. But though they have been known to us for many centuries, the precise nature of the mechanism concerned in these actions is still unknown to us, and has not been satisfactorily explained. What kind of mechanism must that be? We cannot help wondering when we observe two magnets attracting and repelling each other with a force of hundreds of pounds with apparently nothing between them. We have in our commercial dynamos magnets capable of sustaining in mid-air tons of weight. But what are even these forces acting between magnets when compared with the tremendous attractions and repulsions produced by electrostatic forces, to which there is apparently no limit as to intensity. In lightning discharges bodies are often charged to so high a potential that they are thrown away with inconceivable force and torn asunder or shattered into fragments.

***Nikola Tesla** in a lecture delivered before the Franklin Institute, Philadelphia, February 1893 and before the National Electric Light association, St. Louis, March, 1893.*

Introduction

Maxwell's equations have been around for more than a century now and together with modern mathematics and reformulation by Oliver Heaviside became fundamentally the driving force behind nearly all the applications of electricity that define the 20^{th} century. But science marches on and moving on, at times it can be particularly instructive and productive to return to scientific roots and review long-established theories from new viewpoints. In spite of the astounding success of electromagnetic applications to society, a few dark spots have emerged that point to shortcomings or at minimum areas of insufficient understanding with regard to classical electrodynamics. Some of these errors include the reality of a displacement current which throughout the life of the Maxwell theory has never credibly been demonstrated, or the idea that electric and magnetic fields "create each other" to propagate waves in space, or that there is only "one E field".

Heaviside himself was viewed as a crank and crackpot during most of his career and only began to achieve any recognition in his later life and most of his ideas were boldly ignored and pointedly not implemented by the establishment to punctuate their view of his work. Eventually even the most resistant came around such that the Heaviside formulation of Maxwell's cumbersome expressions became what we today term "Maxwell's equations" even though they are not quaternions and were developed by Heaviside. Such heavy-handed blatant manipulation of credit in science has not disappeared with time. And articles and remarks critical of the Jefimenko work we are about to discuss still abound on the Internet and elsewhere. But obviously that is a problem with *scientists* and not with science.
Another problem with science is that given a new topic of interest showing up, understandably, all the easy problems are done first. While calculations of the 20^{th}

century tended to be pencil and paper affairs, powerful computing machines are already a salient feature of the 21^{st} century, which is currently just getting going. What this means is that solutions to practical problems that formerly required approximations and simplifying assumptions in the theory now are capable of producing exact results. For this reason, complexities such as the effects of retardation, which were well known but largely ignored due to calculation difficulties, can now easily be examined in detail.

In our view the importance of such a re-examination is not the mere production of practical solutions to previously intractable problems, but rather the impact upon the philosophy behind electromagnetic theory in general. Concepts such as the storage of energy and momentum in space would appear to possibly benefit from such a new look at the old classical theory.

In my view it's time to start poking around some dark corners of classical electromagnetic theory. The place to look is not where things are *right* with it but rather to look where there are problems. As the late physics professor A. H. Benade, for whom I worked as a student, once remarked to me, we don't want to be like the guy who lost a quarter over there, but is looking here because the light is better!

Charge

All of electromagnetic theory is based upon a single quantity: *charge*. This quantity has been known since ancient time when Thales of Miletus in 600 BC reported that rubbed pieces of amber attracted bits of straw, but today we still have no real idea what it is. The ancients observed effects whereby certain objects attract or repel one another. If you rub a balloon on your pet cat or rabbit, it is said to acquire a "charge". That "charge" after all these years is still a mysterious quantity. It can create attraction or repulsion and is known to occur only in two distinct types like the clockwise and anti-clockwise hands of a clock. If the "charges" of two objects are of the same type they repel, of different types they attract.

Today, nobody knows exactly what charge is, but we do know a few things about it. We know, for example, that objects become charged because of a transfer of tiny particles called electrons, which actually carry the charge in a discrete amount for each one. When the electrons on one object rub off onto another, they carry their charge with them so that the whole object is said to be "charged". And we know something more. As Feynman put it in his lectures[1]: "*One of the basic laws of physics is that <u>electric</u> <u>charge</u> is <u>indestructible</u>; it is never lost or created. Electric charges can be moved from place to place but never appear from nowhere. We say that <u>charge is conserved</u>*".

While this behavior is more or less observed it may or may not be quite correct. For example we observe that a particle and an anti-particle such as an electron (which has a negative charge) and a positron (which has a positive charge) when brought together produce a process called "annihilation" where the two particles disappear and a photon

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1 Feynman, Leighton, Sands, "The Feynman Lectures on Physics", Addison-Wesley Publishing co. Palo alto Calif, ,II –13-2.

which is a particle of light (with zero charge) takes their place. The photon has no charge or mass, unlike the original particles, but does possess energy. Another process known as "pair production" begins with a photon and suddenly produces both an electron and a positron. So clearly either the two kinds of charge can destroy each other and can be created together in pairs or else if Feynman is correct, light particles actually contain two kinds of charge bound up in them, but unobservable because they cancel each other out. But the fact nevertheless remains that a single positive or negative charge is never created alone by itself nor disappears by itself. So if charge disappears, it takes an equal quantity of positive and negative charge so that total charge in both cases is zero before and after the "disappearance" or in other words "conserved". Thus, the total net charge in the universe must sum to zero at all times, but total charge may or may not be fixed.

And we know something else. When charged objects attract each other, the action takes some time to travel between the objects. In the 19^{th} century it was widely believed that this attraction represented "action at a distance". In other words, instantaneous forces, but it was later established that the actions took place at speeds up to the speed of light, which is very fast and hence led to the false assumption that the action was immediate. The theory of Special Relativity today indicates that no action can take place over a distance faster than the speed of light. Hence, all electromagnetic actions are what is termed "retarded". This means that when a source produces an action, it can travel no faster than the speed of light to the place where the action is observed. That necessitates a delay or "retardation" of the action.

Because of this fact, one can easily see that as Jefimenko puts it:[2] "…*an equation between two or more quantities simultaneous in time, but separated in space cannot represent a causal relation between these quantities*." Causality is the principle in physics that all present phenomena are governed only by past events. This will become important later when we examine, questions such as whether an electric and magnetic field can create each other. Generally speaking, retardation effects have been much neglected in electromagnetics because they have been difficult to calculate.

But what exactly this quantity known as charge might be and how it operates at the subatomic level remains unknown to this day. Nevertheless, this mysterious effect of remote attraction and repulsion alone remains the entire basis of the whole study of classical electromagnetics, which is our subject in this paper.

Current

When charged objects are fixed and unmoving they produce the above-described effects about themselves known as electrostatics. But charged objects also can move. And when charges move they are called electric currents. And these charges can move in a most interesting way. This is when we have equal amounts of positive and negative charges moving in opposite relative directions. This is the situation of an electric current in a plasma. A wire is also neutral having no electrostatic attraction or repulsion due to the

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[2] Jefimenko, Oleg, D. , " Causality, Electromagnetic Induction and Gravitation", Electret Scientific Co. Star city, 2000, P4.

charges (which are considerable) within the wire. The action of the positive charges (atomic nuclei) is canceled by the negative ones (electrons), but that does not mean there are no attraction or repulsion phenomena. In 1820 Oersted discovered that a current could move a compass needle and soon Ampere observed that two current-carrying wires attract or repel each other depending on the direction of the currents. This action takes place in spite of there being no apparent charge on the wires. Two wires with current in the same direction attract each other and with current in opposite directions repel.

Prior to the discovery of the above, electrostatic effects and magnetic attractions were studied separately as independent actions and phenomena, but clearly these new observations indicated some kind of connection. While the attraction and repulsion observed in ordinary magnets seems to be a topic totally divorced from the above discussion of charge, today, it is known that these magnets are actually formed by circulating atomic currents. So far as is known all magnetic fields are created by currents. Magnets attract and repel each other, but remain totally electrically neutral.

Fields

The question then arises at this point, just how does one describe all these attraction and repulsion phenomena? The great idea was to use mathematics known as "field theory" as a descriptor. In short, the forces created about a given source be it charges or currents are modeled with mathematical functions giving the values and directions of those forces throughout space. But the trick that makes it all work is something called the *superposition of fields*.

This experimentally observed fact is that one can separate the fields produced from various charges in various regions of space. For example, one can consider two charges in space that repel each other. It turns out one can imagine that the first charge is putting out an electric field throughout space which produces a force upon the second charge. But at the same time that second charge is *also* producing an electric field that acts upon the first charge with a force. It turns out one can add both effects and the result is the observed mutual repulsion of the two charges and Newton's famous Third Law of action-reaction with a force on each other described by the relation known as Coulomb's Law.

And similarly, we find that currents produce a different kind of field subject to different mathematics, which is termed a magnetic field. And if we have two wires attracting one another this time due to magnetic fields, we can separate the fields into that of the first wire upon the second producing a force on it and the magnetic field of second wire upon the first producing a force upon it using a relation known as the Lorentz Force Law. The net result again is that the sum of these two separate calculations is the attraction of the two wires. Magnetic fields are also subject to superposition.

And finally, one can observe that if one had a certain object in space which consisted of various currents and charges upon it, and out in space were various sources consisting of a number of charged objects and current loops, we would also find that our calculation of the forces experienced by our experimental object could be found by a summation of the forces produced by each source be it electric or magnetic considered separately. This, we add up the effects of the source charges or currents upon various charges and currents of our test object. In other words there is superposition between electric and magnetic fields as well, allowing them to be considered separately and summed later. As we shall see, this is an important point because of the widely held erroneous belief that electric and magnetic fields can "create each other'.

We quote Jefimenko on this important point: [3] *"There is a widespread belief that time-variable electric and magnetic fields can cause each other. The analysis of Maxwell's equations presented above [in his book] does not support this belief. It is true that whenever there exists a time-variable electric field there also exists a time-variable magnetic field… but as we have seen, neither Maxwell's equations nor their solution indicate the existence of causal links between electric and magnetic fields."*

[ ] brackets throughout this paper indicate author's comments added to quotations.

An interesting by-product of the facts presented so far is that electromagnetism presents an amazing redundancy as to methods of producing solutions to situations. Probably the most salient of them is Faraday's law. This law states that the emf induced in a loop is equal to the rate of change of the magnetic flux through the area of that loop. It is very interesting that the loop itself need not be in any magnetic field to have an induced emf. This can be seen in the case of a toroidal transformer where the magnetic field is confined to the region inside the primary winding. Hence, it is clear that this cannot be a case of a direct action of changing magnetic fields creating the electric fields producing the emf in the loop unless one is willing to admit of the bogus "action at a distance" theory. Yet, we shall see that this "incorrect" Faraday approach in most cases gives an answer as correct as the true causal calculation involving electrokinetic fields. But not all cases.[4]

Causal forms of Maxwell's equations show that for transformers it clearly is the changing *current* in the primary winding that is the causal source of the emf in the secondary. Maxwell himself essentially observed this when he said, [5]

"When the number of lines of magnetic induction which pass through the secondary circuit in the positive direction is altered, an electromotive force acts round the circuit, which is <u>measured</u> <u>by</u> the rate of decrease of the magnetic induction through the circuit."

Note well that the Maxwell did not say "caused by" but rather "measured by" [emphasis mine]. This is exactly demonstrative of a redundancy we will be finding throughout electromagnetic phenomena and the calculation of it. There are often several ways to calculate an answer that all give the "correct" values. However these various methods

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[3] Jefimenko, Oleg, D. , " Causality, Electromagnetic Induction and Gravitation", Electret Scientific Co. Star city, 2000, p16.

[4] See Feynman Lectures Op. Cit., Vol II Sec. 17-2 "Exceptions to the 'flux rule' "

[5] Maxwell, James Clerk, " A Treatise on Electricity and Magnetism", Dover edition, Section 531, p 179.

may indeed give widely differing implications if one tries to adduce electromagnetic models such as the location of forces from them.

Action-Reaction

One important observation that comes from the inclusion of retardation in our considerations is the fact that much of electromagnetic action violates Newton's law of action-reaction. In the case of static fields, any rapid changes have been assumed to have time to settle down and stabilize and we find that in this case not only can retardation be ignored, but also for every action there is an equal and opposite reaction. In other words speaking electromagnetically, the force the field sources create upon a test object (charges and currents) will be equal to the forces the test object (charges and currents) create upon the sources of the fields that acted upon it.

But with time-variations all that changes. Imagine two small charges separated by some distance. They each create forces upon each other, either forcing them together or apart. But now let us quickly move one of the charges or change the amount of charge there. The force field (electric field, **E**) our changed object experiences from its surrounding fields does not change.. Those fields just keep coming as they always have, but the new force value that the changed charge creates upon the other takes time to arrive there. In other words there will be a finite period of time when the source charge will not "know" that the other changed charge has changed. During that time, action and reaction forces are not equal.

Jefimenko has actually calculated the field theory showing that this is so.[6] But it is also important to note that this law expressed as conservation of momentum remains true. What can be shown is that momentum is impressed upon a field by the sources, which carries it to the action object. It then acts upon that object creating actual physical motion or momentum of the mass times velocity variety. We will examine here the details of this physics linking motion and momentum carried by fields. Newton's second law holds while his third law does not

What is important here is to realize that the mathematical fields that model our electromagnetic action also represent some mechanism of energy transfer and momentum transfer. Energy and momentum applied in one region of space can travel to another region at up to the speed of light and carry energy and momentum to that new region causing actual motion in it. Since such transmissions can be used to transmit information as well as power, electromagnetic fields find great utility.

Force Fields

Given that our electromagnetic observations are fundamentally attractions and repulsions, our mathematical field model is primarily concerned with force fields at the fundamental level. We have modeled electrostatic interactions as charge producing an electrostatic E

[6] Jefimenko , Causality etc., "Action and Reaction in Electric, Magnetic and Gravitational Fields" , Chapter 4, p.67ff.

field (electric field), which in turn can produce forces upon other neighboring charges. The force those other charges experience is proportional to the amount of charge they possess. In other words:

$$\vec{\boldsymbol{F}} = q\,\vec{\boldsymbol{E}}$$

This is the most fundamental charge interaction and remains true regardless of the type of E field involved. We shall see that there are different types of E fields with quite different properties for each, but they all are force fields and all obey the above vector force equation. They all produce a force on charge in the direction of the electric field that is proportional to the amount of charge at the location.

Furthermore, we've already discussed the fact that currents also produce fields. And these "magnetic" fields also produce forces upon charges but in a quite different manner. In this case a charge under the influence of a magnetic field experiences a force given by the so-called Lorenz equation:

$$\vec{\boldsymbol{F}} = q\,\vec{\boldsymbol{v}} \times \vec{\boldsymbol{B}}$$

Here we notice a couple of salient features. Here the force not only is proportional to the amount of charge present, but also to strength of the magnetic B field as well as to the relative velocity between the charge and the source of the B field, which is represented by **v** in that equation. Velocity, **v,** is that of the charge through the **B** field. And even more strange, the force due to the vector cross-product is produced at right angles to the plane defined by the velocity direction and the direction of the B field. Clearly magnetic effects are quite different from the basic electrostatic actions of charges between themselves.

And there is an even more interesting effect. Since the force in the Lorentz equation depends on several things, we find a remarkable thing when currents are examined. Currents are simply flows of charges. But we've already noted that most currents in materials consist of relative motion between positive and negative charges. Negative electrons are flowing one direction while the positive nuclei while usually not actually moving relative to the observer are nonetheless in relative motion in the opposite direction to the electrons. In plasmas charges actually move in opposite directions.

Looking at how a magnetic field is created from moving charges, we see that if we change the sign of the charge, the direction of the force upon that charge reverses. But that if we also change the direction of charge flow, that changes the direction of the created magnetic field back to the original direction. The bottom line being that both the positive and negative charges in a neutral current produce forces in the SAME direction upon charge or current in a magnetic field and that force is at right angles to the line of flow of the charges.

Similarly, if a neutral wire is moved through a magnetic field such that the velocity is perpendicular to the wire, Force is produced upon the charges in the wire forcing negatives one way and positive one the other, which as we've noted represents an electric current. In fact this is the basis of an electric generator or dynamo.
In the case of an electric motor our force calculation becomes:

$$\vec{F} = i\vec{l} \times \vec{B}$$

In this case "i" represents the value of the current in the wire and the vector **l** represents the length of wire upon which we are measuring the force. Not surprisingly the amount of force found is proportional to the current in the wire and the length of the wire upon which the magnetic field acts. As above the direction of the force is at right angles to both the B field and the direction of the current in the wire. In other words the force acts sideways upon the wire.

So our conclusion is that electric **E** fields and magnetic **B** fields are force fields producing forces upon charges and currents. An electric field produces a force upon charges proportional to its own strength and the amount of charge present, but no force upon currents where negative and positive charges cancel. The force is in the direction of the E field. A magnetic field produces forces upon *both* charges and currents that is proportional to its own strength as well as to either the amount of charge or current present. In the case of charges there must be motion between the **B** field source and the charge for a force to appear. In the case of currents the current flow is the charge motion.

Total Forces.

It should be obvious that if we have an arrangement of charges and currents in space acting as sources of electric and magnetic fields, that when these fields act upon some object in space consisting of charges and currents forces will be produced upon that object which can be calculated separately and then added up to obtain the total force experienced by that object. In other words we can write, as did Jefimenko:[7]

$$\vec{F} = \int \left(\rho \vec{E} + \vec{J} \times \vec{B} \right) dv$$

Where he notes: "*Although we have derived these equations for time-independent fields only, they must remain true for all electric and magnetic fields because these fields are force fields*"

Three E Fields

At this point we must ask just what he means by "all electric and magnetic fields". It turns out there can be more variability here than one might imagine. Recalling the Lorentz equation giving forces arising from charge in relative motion and a magnetic field. What if instead of considering that a force arising at right angles from a magnetic field, we designate it as a new type of E field? In such a case we now define a relationship analogous to other **E** fields where **F** = q**E** rather than the right angles of a magnetic relation. In other words we define $\mathbf{E_{Lorentz}} = (\mathbf{v} \times \mathbf{B})$. To be sure, this new E field is still calculated from the same relationship to the magnetic field and relative

7 Jefimenko, Oleg, D. , "Electricity and Magnetism", Appleton-Century-Crofts, New York, 1966, P 512.

motion, but now it adds right in to the already existing expression for force due to electrostatic fields. Which is to say:

$$\vec{F} = q\left(\vec{E}_{Static} + \vec{E}_{Lorentz}\right)$$

We have rather forced the issue here to be sure, but it's an important thought because there is yet another electric field to be considered.

Faraday Induction

Returning again to the 19th century we find that Faraday discovered yet another electrical fact. He found that if you change the current in one wire it induces a current in a neighboring wire. To make a long story short, what he found was that a time rate of change of current creates or we can say "induces" an electric field in all space about that current. It is that electric field that creates the emf that in turn results in currents in nearby wires if the circuit is complete.

Since this new electric field is created based upon the time rate of change of currents, Jefimenko has termed it an "electrokinetic" **E** field. This is to distinguish it from the ordinary "electrostatic" field where nothing is permitted to change with time. At this point it is seen that we have three quite different "electric fields" all producing forces on our charged test object from charges and currents. Just how different these "electric fields" are can be seen in the following table originally created by Professor Hooper.[8]

It should also be mentioned that a changing current also produces a magnetic field in addition to the usual static magnetic field from unchanging DC currents that one can call the Biot-Savart field. This additional field that results from a time rate of change of current we term the magnetokinetic field by analogy to the induced electrokinetic electric field. These two fields are produced at right angles to each other and in phase when source currents are sinusoidal. Later we shall identify them with the transmission of energy into space as electromagnetic radiation.

Thus, we now have a force field equation for electric fields composed of three terms:

$$\vec{F} = q\left(\vec{E}_{Static} + \vec{E}_{Electrokinetic} + \vec{E}_{Lorentz}\right)$$

And as the following table clearly shows these three electric fields have widely differing properties and hence cannot be a "single" **E** field as has been widely taught in the past. A large hint that something was going on in this regard would be the fact that electrostatic $\mathbf{E}_S$ fields are conservative while electrokinetic $\mathbf{E_K}$ are not. Therefore we (and Professor Hooper) seem to have discovered something that has been more or less swept under the electromagnetic rug for some time.

8 Hooper, W. J. , "New Horizons in Electric, Magnetic and Gravitational Field Theory", Tesla Book Co. Chula Vista Calif. 1974, p. 15.

The Three Electric Fields

Field properties		Electrostatic Field	Electrokinetic Field	Lorentz Field
Force on Test Charge		$F = qE_e$	$F=qE_k$	$F=qE_L$
Curl E		= 0 always	= - dB/dt	$= \nabla \times (v \times B)$
Div E		$= \rho$	Always = 0	Always = 0
Inverse square law		Yes	No	Local action
Field Dependence		Depends only on charges	Depends only on currents	Depends on velocity and another field
Potential Function		$E_e = - \nabla\Phi$	$E_k = - dA/dt$	May or may not have potential function
Relation to charges		Charges within field distort field	Charges within field do not distort field	Charges within field do not distort field
Current carrying conductors		Always have a surface charge	Can drive a current without a potential drop along a wire.	Can drive a current without a potential drop along a wire.
Integral of E along a path from a to b.		Value does not depend upon the path.	Value depends on the path chosen.	Value does not depend on path *only* if B is perfectly uniform. In general this is not true.
Poisson's Law with respect to interior of conductors		Obeyed	Not Obeyed	Not Obeyed
Integral of E around a closed loop		= 0 always	≠0 in general	≠ 0 in general
Shielding		Can readily be shielded with conducting material	Can be shielded with sufficient thickness of conducting material	Immune to shielding
Spatial Nature		Continuous throughout all space	Continuous throughout all space	Only present at moving charge
Assumed Spatially distributed energy		$\kappa\varepsilon_o E_e^2/2$ per unit volume	$\kappa\varepsilon_o E_k^2/2$ per unit volume	No Spatially Distributed Energy

Again we point out that in spite of the great differences in the characteristics of these three "electric fields", because they are force fields the forces from each can be summed which allows charge to be factored out of each expression which in turn allows these fields to be summed in calculations as if they were one thing.

But in our case here we are going at this point to follow Jefimenko who only sums the electrostatic and electrokinetic fields and treats the Lorentz forces as motional effects. This treatment then allows one to obtain the causal expressions for **E** and **H** fields as given below. A close examination of these expressions shows that both **E** and **H** fields have either charges or currents as their source. Neither **E** nor **H** has the other field values in their source equations. Hence, it is clear that **E** and **H** fields *cannot* "create each other" in spite of the common belief that this is true.

Jefimenko Causal Source Equations for E and H

These causal Maxwell equations will simply be presented here without their derivation for our discussion of electric and magnetic fields. In short, the derivation that can be found in detail in Jefimenko's books begins by finding the solution to Maxwell's equations[9] and then removing the spatial derivatives from those solutions. If one does that there are obtained the following causal equations for both the **E** and **H** fields.

The Causal Equations for E and H

$$\vec{E}=\frac{1}{4\pi\varepsilon_o}\int\left(\frac{[\rho]}{r^2}+\frac{1}{rc}\frac{\partial[\rho]}{\partial t}\right)\vec{r}_u\,dv' - \frac{1}{4\pi\varepsilon_o c^2}\int\frac{1}{r}\left[\frac{\partial\vec{J}}{\partial t}\right]dv'$$

and

$$\vec{H}=\frac{1}{4\pi}\int\left(\frac{[\vec{J}]}{r^2}+\frac{1}{rc}\frac{\partial[\vec{J}]}{\partial t}\right)\ x\ \vec{r}_u\ dv'$$

At this point it is important to carefully examine the exact nature of these **E** and **H** fields that are generated by charges and currents. Note that the electrokinetic part of the **E** field generated by the derivative of current density **J** has a character quite different from the static $\mathbf{E_S}$ field created by charges. For one thing the created $\mathbf{E_K}$ field direction is that of the source current, **J**, rather than in the direction $\mathbf{r_u}$ a unit vector along the line between the observer and the source charge or current density.

One of the more instructive things we can do is examine the above equations in the case where no sources are changing with time. In this case all terms with derivatives become zero. The equation for **H** is seen reduced to the well-known Biot-Savart expression for the calculation of a static magnetic field due to a current. The expression for $\mathbf{E_S}$ reduces to something like Coulomb's law. More precisely it is half of Coulomb's law since that law represents the force experienced between two charge distributions. Here it gives the $\mathbf{E_S}$ field at some point where we could place the second charge and multiplying that $\mathbf{E_S}$ field by the second charge magnitude gives the force experienced by that charge under

[9] Jefimenko, Oleg, D. , " Electromagnetic Retardation and Theory of Relativity", Electret Scientific Co. Star city, 2000, see sections 2-1 and 2-2.

Coulomb's law. A reversed calculation using the second charges as an $\mathbf{E_S}$ field source and calculating the force on the original charges completes the Coulomb force calculation. A very important point here is that these two well-known electromagnetic laws are *not* universal and are only valid when sources are "static" or in other words, unchanging in time and time derivatives are zero.

Another instructive examination would be to consider the case where sources are only currents. This is a common situation where electricity is flowing on metal conductors, which could be wires wound into solenoids or other shapes or metal antennas. Since in metal conductors positive and negative charges are more or less balanced, we can set charge density, ρ, in the above equations to zero. In such a case, only the last term in the equation for **E** remains, which is an electrokinetic **E** field because it is due to a time rate of change of current density and because it is in the direction of the source current.

It is interesting that both the electrokinetic $\mathbf{E_K}$ field and it's sister magnetokinetic $\mathbf{H_K}$ field fall off as only inverse distance from source currents, rather than reciprocal distance squared as it the case with Coulomb and Biot-Savart static laws. This means that at some distance in space one is left with only the kinetic fields significant, which in antenna theory is typically termed the "far-field" of the antenna. The faster decaying terms then contribute to the so-called "near-field" of the antenna. In antennas the near-field is seen as contributing to the reactive (inductive or capacitive) driving impedance of the antenna which is to say relates to stored energy due to the structure, while the far-field is resistive, representing an energy loss which obviously is the energy transmitted out into space.

Further proof that the kinetic fields above represent the far-field radiation of an antenna can be seen by taking the ratio of the magnetokinetic field term above divided by the electrokinetic field term and noting that:

$$\mu_o \varepsilon_o = \frac{1}{c^2} \quad \text{and} \quad \mathbf{B} = \mu_o \mathbf{H}$$

We find that:

$$\frac{\mathbf{E}_{Kx}}{\mathbf{B}_{Ky}} = \mathbf{c}$$

Which is of particular interest because if one calculates the ratio of $\mathbf{E_x}/\mathbf{B_y}$ in the far field of an ideal TEM plane wave, it is found to exactly equal the speed of light, c. At great distances expanding spherical wavefronts approximate the non-physical TEM plane wave. Where z_o is termed the intrinsic impedance of free space and equals 377 Ohms.

$$\frac{\mathbf{E}_X}{\mathbf{H}_Y} = \mathbf{z}_o = \frac{\mu_o}{c} = \sqrt{\frac{\mu_o^2}{\mu_o \varepsilon_o}} = \sqrt{\frac{\mu_o}{\varepsilon_o}}$$

At this point we see that a great deal of electromagnetic phenomena are directly obvious from the causal expression for electric and magnetic fields. These include static fields, electromagnetic waves and Faraday induction in the case where a variable source current induces a potential or current in a nearby secondary circuit (transformer action). What has not been explained is what happened to Lorentz **E** fields and how moving magnets induce currents in wires. That seemingly appears missing from the causal field equations.

Faraday Induction by Moving Currents

One remarkable feature we observed in the causal equations for electric and magnetic fields is that our third electric field, the Lorentz **E** field, does not seem to appear anywhere, but we shall see that it is simply hidden, but not gone. If we have a conductor (eg. wire) bearing an electric current we know that it can create both electric and magnetic fields. We also are aware that the Lorentz relation depends on a relative velocity between source and observer to produce a non-zero result. So therefore, we may ask the question: What happens if we use our causal equations where the current source is given some constant velocity?

Since we are talking about a metallic conductor here, the source charges all cancel so that only the electrokinetic term for the electric field in our above causal equation remains.

Therefore, we simply give our current source a constant velocity and insert it into the equation. A moving current density at location x', y', z' in space can be written:[10]

$$\mathbf{J} = \mathbf{J}(x' - v_x t,\ y' - v_y t,\ z' - v_z t)$$

Which becomes:

$$\partial \mathbf{J}/\partial t = -\, v_x\, \partial \mathbf{J}/\partial x' \ - v_y\, \partial \mathbf{J}/\partial y' - v_z\, \partial \mathbf{J}/\partial z' = -(\mathbf{v} \cdot \nabla')\mathbf{J}$$

Substituting this value into the electrokinetic term in the Causal Equation for **E** and given that there is only a current so all charge densities are zero, we obtain the relation:

$$\vec{E}_k = \frac{\mu_o}{4\pi} \int \frac{(\vec{v} \bullet \nabla')\vec{J}}{r}\, dv'$$

The spatial derivative can be eliminated using the following vector identity:

$$\nabla'(\vec{v} \cdot \vec{J}) = (\vec{v} \cdot \nabla')\vec{J} + \vec{v} \times (\nabla' \times \vec{J}) + (\vec{J} \cdot \nabla')\vec{v} + \vec{J} \times (\nabla' \times \vec{v})$$

or

$$(\vec{v} \cdot \nabla')\vec{J} = \nabla'(\vec{v} \cdot \vec{J}) - \vec{v} \times (\nabla' \times \vec{J})$$

Since **v** is a constant vector that drops the differentiating terms.

$$\vec{E}_K = \frac{\mu_o}{4\pi} \int \frac{[\nabla'(\vec{v} \cdot \vec{J})]}{r}\, dv' \ - \ \frac{\mu_o}{4\pi} \int \frac{[\vec{v} \times (\nabla' \times \vec{J})]}{r}\, dv'$$

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10 See Jefimenko, "Causality Electromagnetic Induction and Gravitation " p33ff.

As mentioned before the Jefimenko calculations rely on the solution to Maxwell's Equations for fields in a vacuum.[11] For the magnetic Field we find the solution

$$\vec{H}=\frac{1}{4\pi}\int\frac{\left[\nabla'\times\vec{J}\right]}{r}dv'$$

And given that in free space **B** = μ_o**H** we have:

$$\vec{B}=\frac{\mu_o}{4\pi}\int\frac{\left[\nabla'\times\vec{J}\right]}{r}dv'$$

Which since **v** is a constant vector allows the last term in $\mathbf{E_K}$ to be written in terms of **B** and **v** or:

$$\overrightarrow{E_K}=\frac{\mu_o}{4\pi}\int\frac{\left[\nabla'\left(\vec{v}\cdot\vec{J}\right)\right]}{r}dv' - \vec{v}\times\vec{B}$$

Before we go on, we would like to discuss these results. Since **J** is moving rather than time-varying, it seems that this field should be called a Lorentz field rather than an "electrokinetic" one. The **v** x **B** term helps indicate this identification. In fact, were the first integral zero, it is seen that the resultant equation:

$$\overrightarrow{E_L}=-\vec{v}\times\vec{B}$$

would be the Lorentz field except for the sign and we would point out the $\mathbf{E_L}$ here is measured in the fixed frame with the current moving as opposed to the usual expression for Lorentz $\mathbf{E_L^*}$ where the field is being measured in the moving system.

It can be easily seen that in the usual "motor-generator" case where wire velocity is perpendicular to the current flow in it, the dot product of current density and velocity is zero which renders the first term for $\mathbf{E_k}$ zero and indeed the only remaining **E** field is obviously the Lorentz E field which has appeared simply by inserting motion into the causal equation.

In the above equation for the velocity of the current, we have three, x, y and z components. We can assume the current density **J** is in the z direction and we can for the moment ignore the effects of motion in that direction (it will be shown later that it creates a line of apparent static charges) Therefore the velocity components in the x and y directions are perpendicular to **J** and thus render the above integral zero and the Lorentz field equation results. Note that as shown in Figure 1 the direction of the $\mathbf{E_K}$ field is parallel to the current like electrokinetic fields created from changing currents rather than from moving currents.

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[11] A derivation of these retarded solutions can be found in Jefimenko's book: Causality Electromagnetic Induction and Gravitation, Appendix 2, "Derivation of Some Retarded Integrals" and in Jefimenko's book " Electromagnetic Retardation and Theory of Relativity", pp 6-10, and his Electromagnetic Textbook: "Electricity and Magnetism", 2nd Ed. , pp. 46-52.

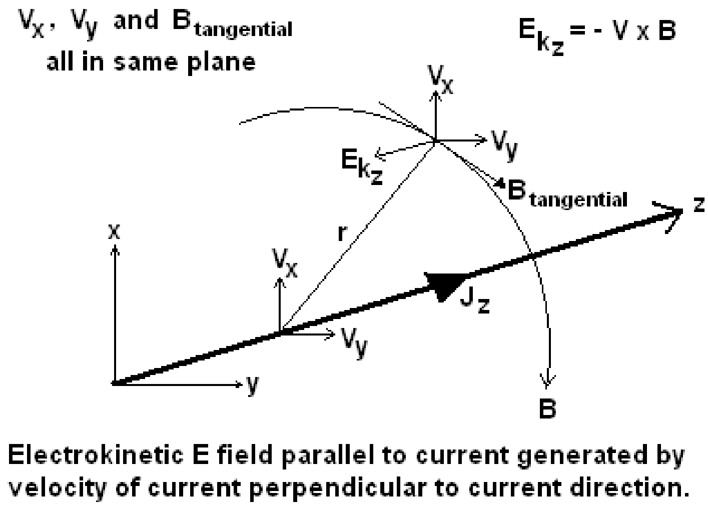

Figure 1. Electrokinetic fields due to current velocity.

To demonstrate the redundancy of electromagnetic calculations, we note how this calculation electrokinetic force is identical to the more usual Lorentz force where a test charge moves through a magnetic field. The only difference here is the negative sign which arises because here the velocity represents current motion which means that the magnetic field is moving over the test charge rather than the test charge moving through the magnetic field. Both give identical results for forces on the test charge.

There may be the temptation here when considering the above mechanisms to attempt to extrapolate to a model for the mysterious electrokinetic electric field that appears parallel to any changing current element. One can try to imagine that an increasing current somehow "throws" a magnetic field out into space with it moving outward as the current is increasing and moving back inward when the current is decreasing therefore creating a Lorentz electric field about that source. This model almost seems to work as an explanation at right angles to the current element. Unfortunately, life is not so simple. The actual electrokinetic field appears parallel to the source current element in all directions about it including along the axis of the source current (straight wires *do* have self-inductance) where the created magnetic field is zero. Thus, any "expanding Lorentz field" theory is clearly inadequate to explain ordinary electrokinetic fields from changing currents.

Faraday Induction from Moving Magnets

At this point one must ask what happens in the case of source current motion in a more general sense where the first term is not zero.

Jefimenko therefore tackles the case of electric fields created from moving magnets. The main feature of magnets is that magnetic fields are created by tiny circulating currents in the atoms of the magnetic material. These tiny circulating currents represent microscopic magnetic dipoles.

We won't repeat Jefimenko's derivation here[12], but it can be shown that motion of magnetic dipoles creates apparent electric dipole fields and that those fields exactly cancel the first term in our expression for the electrokinetic field due to a moving source. So Jefimenko writes:

$$\vec{E} = \vec{E}_{Dipole} + \vec{E}_k = -\vec{v} \times \vec{B}$$

Which clearly shows that in this case:

$$\vec{E} = \vec{E}_L = -\vec{v} \times \vec{B}$$

Velocity in the Direction of Current Flow

In the above cases, the first term in our expression for $\mathbf{E_k}$ for a source current moving with constant velocity was either zero or canceled out. In general, this obviously will not be the case. The question therefore remains as to what would be the effect of source motion in the direction of, or opposite to, the direction of the source current? Jefimenko derives that the effect is to create an apparent line of ordinary charge along the source current creating an apparent "ordinary" electrostatic field.

This "electrostatic" field comes from the first integral in our causal expression for an electric **E** field from which the usual electrostatic $\mathbf{E_S}$ fields are calculated from the charges and not from the second integral, which is usually identified with the electrokinetic $\mathbf{E_K}$ field.

Jefimenko observes:[13]

"Like any electric field, the electrokinetic field of a moving current exerts forces on electric charges located in this field. However, a moving electric current can create not only an electrokinetic field [and obviously a Lorentz $\boldsymbol{E_L}$ *field], but also an "ordinary" [electrostatic] electric field given by the first integral of [the causal expression for* ***E*****]...**[14] *This is because a current carrying conductor moving in the direction of the current or in the direction opposite to the current appears to acquire additional electric charges in consequence of its motion. In the literature this is erroneously considered to be a relativistic effect. Actually, however, this effect is a consequence of retardation and is explainable on the basis of [the causal equations for an E field]".*[15]

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[12] See Jefimenko, "Causality Electromagnetic Induction and Gravitation " p35ff.
[13] See Jefimenko, "Causality Electromagnetic Induction and Gravitation " p34.
[14] Jefimenko observes that the derivative $\partial[\rho]/\partial t$ is associated with electric current through the "continuity relation":
[15] See, for example, W. K. H. Panofsky and M. Philips, "Classical Electricity and Magnetism", 2nd ed., Addison-Wesley, Reading Mass. 1962, pp. 332-334.

Mathematical proof of Jefimenko's assertions and the derivation of the apparent line charge due to motion are given in appendix 3 of his book.[16] This derivation involves some interesting examinations. The first is that the electrostatic field portion of the causal field equations given above, which is to say the part of the total **E** field arising from charges and the time rate of change of charge can be expressed in terms of a retarded scalar potential.

$$\vec{E}_{static} = \frac{1}{4\pi\varepsilon_o}\int\left(\frac{[\rho]}{r^2}+\frac{1}{rc}\frac{\partial[\rho]}{\partial t}\right)\vec{r_u}\,dv' = -\nabla\varphi *$$

Where φ* refers to the retarded scalar potential. Later we shall see that the electrokinetic portion of the total **E** field is related to the retarded magnetic vector potential.[17] This allows the causal equation for the total **E** field to be written in terms of potentials as:

$$\vec{E}_{total} = -\nabla\varphi * - \frac{dA}{dt}$$

Where A* is the retarded magnetic vector potential. However our interest here is only in the Electrostatic field.

Jefimenko derives the *Liénard-Wiechert* potentials, which express the potentials of a moving point charge in terms of the retarded position of the charge. If the charge moves with constant velocity (as we assume here in our problem of a moving current) those equations can be converted to the present position of the charge. Jefimenko does this obtaining for the scalar potential:[18]

$$\varphi * = \frac{q}{4\pi\varepsilon_o r_o\left[1-\left(\frac{v^2}{c^2}\right)\sin^2\theta\right]^{\frac{1}{2}}}$$

where r_o is the magnitude of the present position radius vector and θ is the angle between vectors $\mathbf{r_o}$ and **v.** For the non-relativistic case where v << c that formula can be written:

$$\varphi * = \frac{q}{4\pi\varepsilon_o r}\left(1+\frac{v^2}{2c^2}\sin^2\theta\right)$$

At this point we now consider a short piece of neutral current-carrying wire at rest on the x axis with the midpoint at the origin. We assume that the line density of positive charges is q/L and of negative charges -q/L. If the current is due to the motion of both positive and negative charges moving with velocity **u**, where **u** << c, the current in the wire is given by:

16 See Jefimenko, "Causality Electromagnetic Induction and Gravitation " Appendix 3, p168ff.
17 See for example D. A. Griffiths and M. A. Heald, "Time-dependent Generalizations of the Biot-Savart and Coulomb laws", Am. J. Phys., **59**, 111-117, 1991.
18 Jefimenko, "Electromagnetic Retardation and the Theory of Relativity, p. 96 also p. 55.

$$I = 2\lambda u = \frac{2\text{qu}}{L}$$

An observer on the Z axis at a distance r >>L from the wire (such that sin θ = 1) gives a potential from the positive charges of:

$$\varphi + = \frac{q}{4\pi\varepsilon_o r}\left(1 + \frac{(+u)^2}{2c^2}\right)$$

And the negative of

$$\varphi - = -\frac{q}{4\pi\varepsilon_o r}\left(1 + \frac{(-u)^2}{2c^2}\right)$$

Thus, as expected the resulting potential is zero. But when the wire moves in the direction of the current those equations become:

$$\varphi + = \frac{q}{4\pi\varepsilon_o r}\left(1 + \frac{(v+u)^2}{2c^2}\right)$$

$$\varphi - = -\frac{q}{4\pi\varepsilon_o r}\left(1 + \frac{(v-u)^2}{2c^2}\right)$$

Which gives a total potential from the wire:

$$\varphi = \varphi + + \varphi - = \frac{q}{4\pi\varepsilon_o r}\left(\frac{2\text{uv}}{c^2}\right)$$

Thus, in a kind of analog to the fact that a moving charge produces an apparent current, a moving current produces what is perceived as a charge. Hence the apparent charge on a wire moving in the direction of the current is:

$$q^{+}_{apparent} = \frac{2\text{quv}}{c^2} = \frac{ILv}{c^2}$$

and q is negative for a wire velocity opposite to the current flow.

$$q^{-}_{apparent} = -\frac{2\text{quv}}{c^2} = -\frac{ILv}{c^2}$$

Remember that I is the current, L is the length of the current element, v the velocity of the wire movement, and q the total apparent charge that appears on the wire element. Note that in spite of the value being quite small except at very high velocities, this derivation assumes that v << c so there are no relativistic effects in this solution. The appearance of this apparent line of charge is purely a classical electromagnetic effect.

That a moving charge or a moving observer relative to the charge produces an apparent current (and resultant magnetic field) and a moving current or moving observer relative to the current produces an apparent charge (and resultant electrostatic field) provides us with a philosophical suggestion staring us in the face. And the Lorentz relations showing that electric and magnetic fields change value according to the motion of the observer further reinforce that. Add to that the fact that we found the Lorentz $\mathbf{E_L}$ field "hidden" in the causal electrokinetic term that appeared once motion was introduced.
These effects have not gone unnoticed and has led scientists and engineers to take the view that electric and magnetic fields are really fundamentally the same thing and that there are not just electric and magnetic effects, but only "electromagnetism". We observe, however, that electric and magnetic fields have widely differing properties, which strongly implies that they are *not* the "same thing". On the other hand, our observations showing alterations of these force fields by simply relative motion gives in our opinion the philosophical suggestion that charge and all the effects it produces is at some fundamental level related to a model where charge consists of *motion*!

An Underlying Mechanism

An important speculation that can arise from the above mathematical machinations is that while it seems that electric fields clearly seem to appear in three distinct types, the causal equation for electric fields are able to produce each type of field depending on the conditions imposed. This strongly suggests that there might be some singular underlying fundamental mechanism that the three electric fields are reflecting according to conditions.

In fact, we shall see that motion creates an exchange of values between even magnetic and electric fields which is even more suggestive of a unitary unknown underlying mechanism or at least a model of these field relationships. Back in the early days of electromagnetics the model that was suggested to men of science was that of some manner of energy transmitting, stress transmitting, medium. They termed this unknown and unmeasured medium the "luminiferous aether".

Maxwell made the following insightful comments: [19]

"Now we are unable to conceive of propagation in time, except either as the flight of a material substance through space or as the propagation of a condition of motion or stress in a medium already existing in space."

In other words, Maxwell could think of no mechanism for energy transfer through space other than either the kinetic energy of projectiles or by waves in a medium. So far as is known no other mechanism has been demonstrated. Nevertheless, in a strenuous push to

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19 Maxwell, James Clerk, "A Treatise on Electricity and Magnetism", Dover Edition, Volume II, section 866 p492.

eliminate the concept of a medium from physics (probably having a political basis at some fundamental level) it is currently maintained that essentially such energy transfers occur by magic.

The popular freshman physics textbook Halliday and Resnick opining:[20]

"It is necessary to have a medium for the transmission of mechanical waves. No medium is required for the transmission of electromagnetic waves, light passing freely, for example, through the vacuum of outer space from the stars."

If the best students have trouble with this total contradiction of meaning in the English language it's small wonder. Einstein, on the other hand took a different view than that typically ascribed to him:

"There are weighty arguments to be adduced in favor of the aether hypothesis. To deny the aether is ultimately to assume that physical space has no physical qualities whatever. The fundamental facts of mechanics do not harmonize with this view...According to the General Theory of Relativity, space is endowed with physical qualities; in this sense, therefore, there exists an aether. According to the General Theory of Relativity space without aether is unthinkable."[21]

And "empty" space does indeed have undeniable physical qualities. The most salient of these are $\square_{o}$, the permittivity of free space and $\square_o$ the permeability of free space. Together these form the intrinsic impedance of free space given by

$$Z_o = \left(\frac{\varepsilon_o}{\mu_o}\right)^{\frac{1}{2}} = 377.7 Ohms$$

Which as Einstein pointed out is unarguably a property of "empty" space.

"This spacetime variability of the reciprocal relations of the standards of space and time, or, perhaps, the recognition of the fact that "empty space" in its physical relation is neither homogeneous nor isotropic, compelling us to describe its state by ten functions (the gravitation potentials $g_{\square,\square}$ *), has, I think, finally disposed of the view that space is physically empty."*

However, it is to be stressed that there is no 19th century aether model being promoted here nor is one even being suggested. The purpose of this discussion is to point out the

20 Halliday and Resnick, "Physics for Students of Science and Engineering", John Wiley, 1960, Vol IU p. 393.

21 . Albert Einstein, From a speech given in German May 5, 1920 at the University of Leiden, Holland, translated into English by Sir Oliver Lodge and quoted in his book, "Either and Relativity" (1925). Quoted here from "The manual of Free Energy Devices and Systems" Vol II by D. A. Kelley, 1986, 2nd printing by Cadake Industries and Copple House, Clayton Ga, (1987).

strong indications merely from the mathematical interrelationships of electric and magnetic fields that some manner of underlying model could possibly be conceived that might act as an aid to the understanding of the complexities of electromagnetics.

Lorentz Transforms

With regard to measurements of moving systems, Jefimenko notes:[22]

"*The principle of relativity was first enunciated in 1632 by Galileo as a statement of the fact that there are no experiments or observations whereby one could distinguish the state of uniform motion along a straight line from the state of rest. However, in accordance with the level of scientific knowledge of his times, Galileo supported this statement by citing only mechanical experiments with an indirect reference to the laws of optics. At the beginning of the 20th century Lorentz, Poincaré, Larmor, and Einstein, in separate works demonstrated that the principle of relativity was applicable to electromagnetic phenomena as well.*"

In these discussions, one often refers to a Laboratory or "Laboratory frame". A laboratory means where the measurements are being made. A "frame" means the collection of objects which all move together. Often the term "reference frame" is used which means "laboratory frame". Typically, but not always, the laboratory frame is assumed stationary. The moving frame is called the "inertial frame" which means a collection of objects (which could be measuring devices) moving together in a straight line at constant velocity. For example an inertial laboratory frame moving at constant velocity v could be a flatcar loaded with measuring instruments, traveling down a straight track.

While we've just noted that if the electric and magnetic field sources are moving with a constant velocity in a straight line and the observer measuring those fields is *also* moving in the same frame one has no way to determine if the system is moving or not.

However, when the electric or magnetic field sources are in one frame and an observer moving at a different velocity in an inertial laboratory frame makes the measurements, the situation is quite different. In the moving measurement frame the instruments may read different values for the **E** and **H** fields measures when the instruments were at rest in the stationary source frame. Experiment has shown the **E*** and **H*** fields, where the * indicates a measurement taken in a frame moving with velocity v with respect to the stationary source frame, are given by:

$$\vec{E}* = \vec{E} + \vec{v} \times \vec{B} \quad \textbf{and} \quad \vec{H}* = \vec{H} - \vec{v} \times \vec{D}$$

What is observed is that once the field meters are moving they begin to measure additional "Lorentz fields" that were not measured when they were stationary. And we observe that the added Lorentz *electric* field is of the Lorentz type we discussed above. In other words, the added field is dependent upon velocity and a magnetic field to exist.

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[22] Jefimenko, Olg, D. , " Electromagnetic Retardation and the Theory of Relativity", Electret Press, 2nd ed. 2004, p 130.

Is such a thing reasonable? How can it exist? Consider for a minute a particle in space with a certain amount of charge, q, on it. That object will produce a radial electrostatic E field. If we use a charged "test" particle to probe the field about that source, we observe the radial forces to the source and hence measure $\mathbf{E_S}$. Then let us imagine the same situation only now our "meter" is traveling on a flat car. To make it easier consider that the meter is stationary and the charged source object is moving away to our left when we face it. When that object is momentarily right in front of us it is still producing a radial electrostatic $\mathbf{E_S}$ field. So we measure that. But there is more. Since the charged object is moving to our left it clearly constitutes an electric current. And as we know an electric current (even a fictional one like this) produces a magnetic field. That field will be vertical at the meter. And by the law, $\mathbf{F} = q\mathbf{v} \times \mathbf{B}$, our test particle will experience a Lorentz force at right angles to the direction of motion. Which can obviously be written as a Lorentz **E** field as seen in the above transforms. So the "new" fields neither defy Galiean relativity nor common sense. But they are *not* "relativistic" in the Einsteinian sense because the velocity is much less than the speed of light.

To justify the magnetic transform, consider the same situation outlined above. In the stationary frame when we measure the magnetic field from the stationary charged source object, we obviously find none as it's only stationary charge. But measuring in the moving frame that charge now constitutes a current and the same magnetic field which produced the Lorentz E field above appears as the "new" magnetic field in the second transform relationship. So it is clear that all these apparently changing field values are really arising because the Lorentz force equation. Note however these transforms are only approximate as the electrostatic line charge due to motion in the direction of current calculated above does not appear in these transforms.

The only problem is determining what kind of physical model could be invented to "explain" the creation of new fields merely by velocity which we have already observed in our examination of the causal equations for electric and magnetic fields. All this creation and transformation of electric and magnetic fields strongly suggests that both are somehow intrinsically linked at some underlying fundamental level.

Energy Storage in Space

We have already discussed the "modern" nonsensical doublethink idea that electromagnetic waves propagate in nothing at all. Maxwell didn't accept such an idea for a second as it flies in the face of the very definition of what wave propagation is defined to mean. Wave propagation by definition and as Maxwell understood it, means that one has a medium of some sort that is fixed. Energy applied to that medium stresses it at that point. Energy propagation then indicates that those stresses somehow by the very fundamental interconnections of the essence of the medium over a distance between locations in it will somehow transmit that stress outward in repeated fashion eventually allowing that initial energy to spread throughout the entire medium. The medium stays in place but the energy is transmitted over distances.

Since Maxwell pointed out that only a medium or projectiles can transmit energy, and he concluded since electric and magnetic fields transmit energy (clearly as they produce forces at a distance) they therefore must be either particles or what he termed “medium in a state of stress”.[23] He opted for the latter hypothesis proposed earlier by Faraday of a stressed medium much like energy stored in an array of springs. From this hypothesis Maxwell concluded that the total energy stored in space represented by an electrostatic field in free space is given by:

$$U = k\frac{\varepsilon_o}{2}\int_{All\ Space} E^2 dv$$

Here ε_o is the permittivity of free space and k is the energy proportionality constant which is typically set to unity by a careful choice of units and ignoring the fact that it is not dimensionless. Since $\varepsilon_o\mathbf{E} = \mathbf{D}$ in free space, the above equation is often cleverly rewritten in the form:

$$U = \frac{1}{2}\int_{All\ space} \vec{\boldsymbol{E}}\cdot\vec{\boldsymbol{D}}\ dv$$

Similar arguments hold for magnetostatic fields where total energy is often expressed as:

$$U = \frac{1}{2}\int_{All\ Space} H\cdot B\ dv$$

But the expressions are very misleading since they really represent energy stored in free space where $\mu_o\mathbf{H} = \mathbf{B}$, and $\varepsilon_o\mathbf{E} = \mathbf{D}$ rather implying the expression will work for fields in media. However, in magnetic media where μ_o becomes μ the latter may be quite non-linear. Experiment determines the values. Thus, the best and least confusing expressions for the total energy stored in Electric and Magnetic fields are given by:

$$U = \frac{1}{2}\int_{All\ Space} \varepsilon\ E^2\ dv$$

and

$$U = \frac{1}{2}\int_{All\ Space} \mu\ H^2\ dv$$

A natural leap of faith is often made from these equations by the assumption that the amount of energy stored by a field is proportional to the square of the static field magnitude so the integral operation can be removed yielding an energy density in space due to the fields at any given location. Note that neither field here is changing with time.

This makes an important point. When one has a field device such as an inductor or capacitor, which are well known for storage of energy, it is seen that according to theory

[23] Maxwell, James Clerk, “A Treatise on Electricity and Magnetism”, Dover Edition, Volume II, section 641 p278.

the energy is actually stored as stresses in *space itself* and not in the metal, currents or other materials of the devices. The geometry of the conductive metal is just there to create the fields in space, which implies stresses in what Maxwell imagined as a spring-like medium. This can explain how it is that inductance and capacitance of devices are found only to depend upon geometry. Then, later, those stresses can be relieved through a relaxation of the stresses that returns the energy to the electric currents of the devices.

While the above calculations are integrals over all space that compute the total energy that is stored in source-free space represented by the fields, which is important for energy determinations in inductors and capacitors, one can also simply define a small region of space and integrate over that limited volume(s) to find how energy is proportionately distributed in various locations in space.

We note that the above equations allow us to determine the amount of energy stored in source-free space once a field is present there but says nothing about the flow of energies into and out of space. Maxwell observed[24] that energy is always zero or a positive value unlike vector quantities like momentum or velocity. The squared fields in the above equations achieve that end. The important result is that when you have two opposing fields in space, unlike superposition where one might assume each field would have a positive energy that would add, instead it is found that the fields must be combined *first* and *then* the stored energy computed from the resultant total field. For energy, field superposition does not work.

Hence one must think of electric and magnetic fields representing stresses applied to space. But when fields are equal and in opposite directions, one must assume that the "stress" created by the first field is essentially "undone" by the oppositely directed field. Hence a magnetic field in one direction creates stresses in space, but an equal and oppositely directed field removes the stress yielding no stored energy even though relative motion of the fields can show that they still exist because Lorentz forces are created. (See Fig. 13) That non-inductive bifilar coils store little energy demonstrates that equal and opposite magnetic fields are not stressing space in regions where fields cancel.

It is also interesting to note that orthogonal fields do not interact and each can store energy independently of the other. Imagine sets of parallel plates in a vacuum in the form of an approximate cube but not touching each other. This forms three capacitors that are independent of each other and each can store energy in the space within the cube by virtue of the fields existing there. The same goes for three orthogonal straight wires in space storing energy by virtue of their inductance. In each case the same volume of space holds triple the energy of the single device. Notice that the Pythagorean theorem states that sum of the energy stored by each orthogonal component is equal to the energy calculated for the resultant vector of those components.

[24] Maxwell, James Clerk,"Treatise on Electricity and Magneticsm", Dover, Vol II, Section 566,

The Medium of “Empty” Space

Following the Maxwellian theory of energy stored as stresses in the medium of empty space we need to strongly point out that the philosophy here is that mathematical field theory *describes* the phenomena of electromagnetic forces and energies in space. It has become common to say that energy, momentum, angular momentum are “stored in fields” and this is obviously quite wrong. Nothing is stored in the mathematics. Energy is stored in supposed, “stresses” of space that are described more or less by the mathematics over some range of values!

Since we are suggesting consideration of Maxwell's stress theory, we should discuss the nature of this space medium. For lack of a better term it can be called “aether”. It should be emphasized, however that today the known properties of this surmised medium are not at all those assumed in Maxwell's time. We have already shown that the impedance of empty space is one such property. Energy storage is another. The propagation of disturbances at up to the speed of light is another. But in modern times, phenomena unobserved in Maxwell's time begin to explain certain electromagnetic properties.

The phenomena in question are those generally termed today as “super”: superconductivity, superfluidity, and other seemingly lossless effects. There used to be a great debate in the 19th century as to whether aether was very fluid liquid or a rigid solid. Today we would answer: both! On the one hand we find electric and magnetic fields obeying superposition, which implies that they flow past each other, even in opposite directions without effect or interactions, and yet it is also known that electromagnetic waves can be transverse polarized which implies the kind of medium presented by a rigid solid. Hence it becomes clear that this medium is *both* a freely flowing super-fluid and a transverse wave-transmitting solid at the same time. Electromagnetic energy storage in space alone as well as the observed transmission of waves and aether stresses show the largely lossless nature of this all-pervading medium. Classically all these properties would seem to be “doublethink” word games like the previously mentioned waves in “nothing at all”, but in this case the “super” properties in modern times have been observed and studied such effects as super-conductivity and super-fluidity and hence are based on experimental observations.

While today many do seem to think that physics is some kind of semantic word game, but this is not the nature of this science. Dr. Lewis E. Little in his book on his Quantum Theory[25] remarks:

“Physicists conceive of the 'field' as consisting of the force it is capable of exerting. They frequently refer to gravitational, electric and magnetic fields as 'behavior fields'.”

“But how can any object or 'thing' consist of its behavior? The concept of 'behavior' presupposes the existence of the object that behaves. Treating a behavior field as an

[25] Little, Lewis E., “ The Theory of Elementary Waves”, New Classics Library, Gainesville GA, 30503, 2009, p. 105.

object assumes that the behavior can exist "by itself" not as behavior of anything, just behavior – of nothing"

"The magnetic field itself became the 'real' object. Physicists to this day treat electric and magnetic fields as if they were the actual players involved in the phenomena of electricity and magnetism, despite substantial evidence proving that such a picture of things could not possibly be correct."

Here we shall attempt to not fall into this trap and maintain the distinction between observed reality and mathematical models of that reality.

Energy Flow

Energy flow into and out of any defined volume of space can be determined by the following equation, which we won't derive here but is derived in most EM textbooks and is called the *Poynting Theorem:*

$$\oint_S \vec{E} \times \vec{H} \cdot \mathbf{ds} = -\frac{\partial}{\partial t} \int_V \left(\frac{\mu}{2} H^2 + \frac{\varepsilon}{2} E^2 \right) dV - \int_V \sigma E^2 \, dV$$

What this represents is a chosen volume of space where the left side represents the instantaneous flow of power[26] out across the surface that bounds that volume, which is equal to the time rate of decrease (minus sign) of the magnetic and electric energy stored in that volume minus any power loss due to Joulean heating. It is important to notice that while this theorem involves vectors it is a scalar not a vector relationship. The integral over the surface of an arbitrary volume containing **E** and **H** fields is a single value number. The cross product of the **E** and **H** vectors give a vector we'll call **S** that when combined in a dot product with d**s** which is an element of surface represented by a unit vector d**u** perpendicular to the surface at that point.. The result of the dot or scalar product is a simply a single number that when summed over the surface by integration results in a single final value for the term. It is important not to confuse the Poynting vector which we have termed **S** with any vectors representing a surface element **ds.**

The arbitrary volume can enclose sources of electric and magnetic fields, but it is important to remember that these sources, charges and currents, must be described as charge density ρ or current density J. The reason for this is that if one attempts to use point charges or line currents, the distance term, r, appearing in the denominator results in an undefined mathematical operation (division by zero) as one integrates near the source. When densities are used instead as one integrates the source, while distances go to zero the differential volume of charge density also is going to zero and saves the operation.

If we examine the final Joule heating term we notice that it is subtracted and represents a loss of energy. Of course by conservation of energy, this energy is not lost at all. It goes

[26] Note that power is the time rate of energy flow. Kilowatt-hours equal energy. Kilowatts or Watts is power. Kilowatt-hours per hour gives an energy rate of flow and equals Kilowatts or power.

into heating the material with a conductivity σ, which then over time radiates that energy into space until the material again reaches equilibrium temperature with its surroundings. But for the purposes of this theorem it is just considered "disappeared". Observe that the volume integral only produces a value in regions where the conductive material exists. Outside the material, the conductivity equals zero and hence that portion of the volume integration also equals zero regardless of the strength of the **E** field.

The equations for stored electric and magnetic energy are already familiar, but now we are talking about the rate of change of these values, which implies an energy flow either into or out of the given volume of space. In "empty" space which means no sources or other materials save normal vacuum, the resistive heating term will be zero because σ equals zero and μ will equal μ_o and ε will equal ε_o. And finally, we notice that while energy can be stored in space by the presence of either an electric field **E,** or a magnetic field **H,** alone, it is clear that for this energy to either be stored into space or withdrawn from it, *both* simultaneous **E** and **H** fields are required. However, we have already seen that once the energy is stored in space for either field, the other field is not required to be present to maintain the storage. In fact, the absence of the companion field *insures* that the energy remains stored in space!

The vector

$$\vec{\boldsymbol{S}} = \vec{\boldsymbol{E}} \times \vec{\boldsymbol{H}}$$

is called the Poynting vector and is defined related to a flow of energy or power (both magnitude and direction) in or out of a volume of space where energy is stored either magnetically or electrically. Since $c^2 = 1/\varepsilon_o\mu_o$ and $\mathbf{B} = \mu\mathbf{H}$ in free space, we find that the *Poynting vector* can be written as below though care must be exercised in using μ in magnetic materials.

$$\vec{S} = \vec{E} \times \vec{H} = \frac{1}{\mu}\,\vec{E} \times \vec{B} = \varepsilon_o c^2\ \vec{E} \times \vec{B}$$

S is a typical letter used for the Poynting vector to avoid confusion with momentum, power, and similar usages. The relationship of **S** to a flow of energy is easily seen in the case of electromagnetic waves where there is found in phase electric and magnetic field vectors at right angles to each other that are also perpendicular to the direction of propagation. With static fields as we shall see below the relationship to power is not so obvious.

Although an extrapolation of the Poynting vector to represent a power density at a given point in space is logical, we should stress that our defining equation above only states that the total surface integral of **S** gives the net power flow across a *closed* surface that is equal to the change in energy contained within that volume defined by the surface.

The electromagnetic textbook Plonsey and Colin makes this clear in the following manner: [27]

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[27] Plonsey and Colin, "Principles and Applications of Electromagnetic Fields", Mc Graw-Hill, NY, p 307.

"*While the interpretation of **E x H** as representing power density at a point is ordinarily a useful one, it should be noted that [the Poynting Theorem] states only that total surface integral of **E x H** gives a net power flow across a closed surface*."

Feynman addressed this problem in his lectures this way:[28]

"*Before we take up some applications of the Poynting formulas we would like to say we have not really proved them. All we did was find a <u>possible</u> "u" and a <u>possible</u> "S"…There are in fact an infinite number of possibilities for u and S and so far no one has thought of an experimental way to tell which is the right one. People have guessed that the simplest one is probably the correct one, but we must say that we do not know for certain what is the actual location in space of the electromagnetic field energy.*

But Feynman, as he puts it, takes "the easy way out" and just assumes that the Poynting vector represents a power density in space.

There is great redundancy in Maxwellian electromagnetic theory. There are often more than one way to calculate a quantity all of the ways usually giving correct answers, but differing greatly in their philosophy and the conclusions that can be drawn from the models. For example, Faraday's law that a changing magnetic field supposedly creates an electric emf field is widely used and accepted. It usually (but not always) gives the "correct" answers, but it is *not* philosophically correct in that **E** and **H** fields do not "create each other". Similarly we shall see that while the Poynting vector seems to represent well the power density in a propagating wave as well as the momentum carried by that wave, but in certain other cases that we will examine below, while the Poynting Theorem remains correct, the assumption of the Poynting vector representing power density at points in space has serious problems.

DC Energy Flows in Electric Wires

Let us consider the energy flows in that most essential circuit element, the electric wire. If we examine a straight piece of conductive wire, we know quite a bit about it. For one thing using Ampere's law we can find the magnetic field tangent to circles at radius, r, about the axis of a long straight wire of circular cross section of radius, R, when a constant direct current, i, flows in it. Inside the wire the magnetic field is given by:

$$H = \frac{i\,r}{2\pi \mathrm{R}^2}$$

Outside the wire the at a radius r, greater than R the field is given by:

$$H = \frac{i}{2\pi \mathrm{r}}$$

[28] Feynman, Leighton, Sands, "The Feynman Lectures on Physics", Addison-Wesley Co. Palo Alto, 1963, Section 17-4 Volume II.

Two points worthy of mention are that the magnetic field is not uniform inside the wire, but rather rises linearly from zero at the axis to a maximum value at the perimeter. And that field outside the wire falls off rapidly being a small fraction of its maximum value just a few wire diameters away from the wire perimeter. Even more interesting is that it will be seen that while electric and magnetic fields fall off inversely with distance from the wire, the power flow is given by the product of those fields meaning that any power flowing in the space by the wire is concentrated even closer to the conductive material.

The next task is to determine an electric field outside the wire. It is widely asserted because of the voltage drop along a wire carrying a current that the electric field outside the wire is parallel to the wire that is to say parallel to the current flow in the wire. From this supposition, it is asserted that the cross product of the circular magnetic field and the parallel electric field produces a Poynting Vector directed inward toward the wire. This is then said to represent a power flow from the fields in space into the wire give rise to the resistive heating in the wire. But there are a number of problems with this conception. First, is that if the wire is highly conductive (as is common) there will be virtually no voltage drop from one end to the other to create an E field and even worse, if the material is a good conductor boundary conditions require that all E fields enter the surface of the wire perpendicularly. Tangential **E** fields are not allowed.

What is interesting is that if we wish to transport more energy down the wire in a circuit to some other device above and beyond Joule heating, we simply raise the potential of *both* ends of the wire. The Joule heating is only due to current that is related to the small potential difference between the ends of the wire, so it remains largely a function of current, and the material, but the energy transport is a product of the magnetic field produced by that current and the electric field found outside the wire from it to the nearest ground. That field can grow large as the voltage on the wire rises above ground. Hence the advantage of using high voltages for transmission of large amounts of power with minimal losses is easily seen.

So let us ignore the Joule heating problem for a minute and consider what happens if we connect the wire to some high electric potential. In this case the electric field from that potential will be radial about the wire. If we cross that **E** field with the circulating **H** field we find a Poynting vector showing energy transmission parallel to the wire but out in space. From this fact many textbooks breathlessly conclude that no energy is going down the wire, but instead all energy transmitted by the wire is in space about the wire. This is not a totally off the wall idea since in waveguides for example, that seems a valid conclusion. Here, however, we are talking about direct currents and constant voltages that guarantee that the energy stored in space about the wire is not changing with time provided there is no Joulean heating removing energy.

Consider coin shaped volumes about the thin wire. In this case the S vector points down the wire all along it. So the S vector enters one side of the “coin” and leaves the other side. Seemingly transporting energy down the wire. The Poynting Theorem on the other hand calculates the integral of S over the coin shaped volume. With a highly conductive wire the value will be virtually identical on both sides of the “coin”. But that vector is scalar multiplied by the unit surface vector. That vector always points in an outward direction. Thus, the integral of the left side of the volume is the negative of the value of the right side. The contribution from the edges is zero so the total Poynting integral equals zero. Since the transmission is DC, this means that the term involving the time rate of change of **E** and **H** is also zero and since the air about the wire is an insulator the Julian heating term is also zero. Hence *all terms* in the Poynting theorem are *zero* meaning no energy changes with time and hence no excess energy moving in or out of the volume of space.

But what about that energy zipping through the air outside the wire? The argument is that the since S represents a flow of energy, there is a flow of some energy into one side of our coin volume and an equal flow of energy out the other side. Since these flows are equal, the energy found within the volume does not change rendering our above calculation still correct.

A conclusion might be that energy flow down a wire is largely a matter of energy in space as expressed by the fields external to the wire, since the amount of energy transported depends both on the current in the wire and the potential to which it is raised, but Joule heating of the wire is primarily related to fields internal to the wire and the motion of charges, which is to say the current internal to it. Thus, the Poynting theory has provided a theory that says that energy is not transmitted inside a wire (like a garden hose for electricity) but rather somehow as stresses in space outside the wire. Even more interesting is that the energy stored in the space around the wire represented by electric and magnetic fields is not changing at all. There seems to be no energy flow in and out of a volume of space but rather a power flow through space while the energy stored in it remains constant. And as we indicated above there are those who say that even the Joule heating enters the wire as a Poynting vector from the space outside the wire and isn't due to current within the wire. We'll have to look into this further!

Joule Heating

It is widely known that an electric current flowing in a conductor with a given conductivity, σ, produces heating. In circuit theory it is said that applied voltage and current represents a certain power into the material, which by reason of mechanisms within the material becomes converted, to atomic motion called heat energy. Radiant heat sent through space is theorized to be nothing more than electromagnetic radiation of a very high frequency, though lower frequency than light. So the Joule heater is basically a high frequency radio transmitter powered by a steady flow of charge and sending high frequency "broadcasts" into space where they are lost forever.

Since current is a flow of charge, it is clear that it takes an electric field to apply a force to the charge to get it moving into a current. Only once the charge is moving as a current does it create Joule heating in the resistive medium. But we also know that the electric field inside a perfect conductor must be zero in the presence of an electrostatic field. And in addition it is known that electrostatic **E** fields cannot set up steady currents.[29] Clearly what we have run into here is a problem with our three types of **E** fields!

It is known that currents do indeed flow in conductive materials producing infrared (heat) radiation as a result. In fact, current and applied electric fields have been experimentally found to obey the equation:

$$\vec{J} = \sigma \vec{E}$$

Where σ is "*conductivity*" of the material that can be used to get the following relation for Joule heating:

$$\int_{conductors} \vec{J} \cdot \vec{E} \; dv$$

Which leads to what is known as Ohm's law. However, if the **E** field is conservative, an integral around a circuit is zero and eventually the current must stop. Continuous currents require a constant supply of charge to replace that drained away by the conductivity. A conservative field is not capable of giving away energy indefinitely.

"As a matter of fact, if we assume the field to remain unchanged, as must be true if a steady current exists, then an electron making a closed circuit in an electrostatic field gains no net energy from the field." [30]

So a charged capacitor with a piece of conductive material shoved between the plates does produce a current and the material gets hot, but the potential on the capacitor is constantly dropping and eventually becomes zero and the current stops. Steady currents require a constant source of charge and hence non-conservative fields. A battery is one source of charges, as is induction from electrokinetic fields or generation from motion in Lorentz fields. In each case the potential gained by a test charge passing through the non-conservative field is termed "electromotive force" or emf. Since a charge can make many passages around such a circuit, it supplies the needed energy that is constantly leaked away by the conductor. Thus a battery or generator connected to our capacitor with a conductive block between the plates held at some potential does indeed produce Joule heating in the block and the electrostatic field of the capacitor stays unchanged. Thus, the **E** field in the above conductivity equation can indeed be an electrostatic field, but to provide a stationary current there must be a non-conservative field somewhere supplying the charges lost to the conductor.

If we now think about our original Poynting situation, we observe that the Joulian heating term is ONLY present when there is a conductive material present within the volume in question and if the electric fields are of the correct type! Since it is known that all materials at the atomic scale are mostly empty space, it is clear that those **E** fields that can penetrate the conductive material also contribute to the indicated stresses storing

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[29] Plonsey and Colin, "Principles and Applications of Electromagnetic Fields", Mc Graw-Hill, NY, pp164-167.
[30] Ibid.pp167-169.

energy in space. But since electrostatic **E** fields cannot exist inside a good conductor, that type field integrates to zero over the volume of the space and given a volume completely filled with conductor, there can be no electrostatic contribution to the energy stored within the volume. For the most part only electrokinetic and Lorentz electric fields contribute to energy stored in space within conductors. Superconductors can have no electrostatic E fields within the material and hence no Joule heating and no resistance to an electric current. Yet currents flow. Presumably driven by limiting cases of electrokinetic fields with extremely slow time rates of change.

But we must remember that materials are not simply conductors or dielectrics. Material comes in a complete range of conductivities from highly conductive all the way to insulating materials. Hence we can say that while for the most part conductive objects never have tangential electrostatic fields at the surface or internal fields in the material, this is not entirely true because there is no distinct transition between insulators and conductors. Furthermore, tangential electrokinetic and Lorentz electric fields not only exist at the surface of conductors but also are continuous in value across that interface. They are the same inside and outside the conductors.

And furthermore, a voltage applied to a wire from a battery creates a current down that wire which creates a voltage drop meaning that there is variation of potential down the length of the conductor. This also creates a tangential electrostatic **E** field on the outside of the conductor. And it can be shown that a surface space charge can accumulate on electrically inhomogeneous conductors bearing steady currents. This charge is given by:[31]

$$\rho_{charge} = \varepsilon_o \vec{J} \cdot \nabla\left(\frac{\varepsilon}{\sigma}\right)$$

But for this charge to exist either σ or ε or both must vary with position such that $\nabla(\varepsilon/\sigma)$ does not equal zero so this charge cannot be used to “explain” Poynting power flows in DC cases. The point here is not these particular calculations, but rather that the “one E field” dogma promoted by Slepian[32] has caused the issue of three kinds of electric fields to be deftly swept under the rug for some time now.

The bottom line here is that when there are currents in conductive media, calorimetric measurement have shown the dissipation of energy to be:

$$power = \frac{dU}{dt} = \int_{conductors} \vec{\boldsymbol{J}} \cdot \vec{\boldsymbol{E}} \; dv = \int_{conductors} \sigma \; E^2 dv$$

Which is seen to be the last term in our Poynting theorem above.

[31] See Jefimenko, “Electricity and Magnetism”, Section 9.5, p. 295ff.

[32] J. Slepian, “Electrostatic or Electromagnetically Induced Electric Field”, Scientific paper 1451, Westinghouse Research laboratory, 7/18/49.

“There is but one God Allah, And Mohammed is his prophet!

“There is but on electric field E, And Maxwell is his prophet!

Inductance compared to Joule Heating

It is interesting to note that in some textbooks discussing Poynting vectors there is often used the Biot-Savart magnetic field about a wire and a presumed external electric field parallel to the wire that is supposedly related to the internal electric field driving the current and the fact that resistance of the wire creates a voltage drop along it which implies a potential difference and hence an electric field parallel to the wire.[33] Such a calculation thus produces an energy flow from space into the wire from the sides often said to be the energy of Joulean heating which is presented as a surprise that it comes from outside the wire instead of internally. Such presentations are misleading not only because there is little electric field outside a conductor carrying a constant current and what field is there is certainly not uniquely related to the amount of joule heating. This is because unlike the magnetic energy flows of inductance, Joule heating does not depend upon the configuration of the wire. That implies that most of the electric field causing the power flow into the wire for Joule heating is confined to the interior of the wire. An exception would be the case of an electrokinetic non-conservative field as in induction where the driving field is uniform throughout space both inside and outside the wire. In this case the current and hence heating *can* depend on the wire configuration.

Maxwell in comparing the inertial effects of water in a hose to self-inductance in a wire noted[34]

"These effects of the inertia of the fluid in the tube depend solely on the quantity of fluid running through the tube, on its length, and on its section in different parts of its length. They do not depend on anything outside the tube, nor on the form into which the tube may be bent, provided its length remains the same."

"With a wire conveying a current this is not the case, for if a long wire is doubled on itself the effect is very small, if the two parts are separated from each other it is greater, if it is coiled into a helix it is still greater and greatest of all if when coiled, a piece of soft iron is placed inside the coil."

Since we have seen how the storage of magnetic energy and it's return very much depend on the configuration of the source and electrokinetic energy in space about the conductor and since we are all aware that Joule heating does not depend upon the configuration of the source current, the conclusion must inevitably be drawn that the fields giving rise to the energy flows of Joule heating from ordinary currents are not external to the wire as Feynman has assumed. It is from the E field internal to the wire driving the current.

A somewhat different situation where fields clearly *are* external arises in the case of a closed inductive loop about a long solenoid of wire. A time rate of change of the current in the solenoid gives rise to an electrokinetic E field in space about the solenoid, which exists both inside and outside the looped turn outside the solenoid. Since there is little B field outside the long solenoid except for end effects, it is clear that the magnetic field

33 See for example Feynman's lectures volume II section 27-5 , p 27-8.
34 Maxwell, James Clerk, "A Treatise on Electricity and Magnetism", Dover Edition , Section 548, p196.

used to calculate the Poynting vector of power flow into the secondary ring must be generated by the current in the ring itself. This is obvious since breaking the ring stops the induced current and also stops the Joule heating. Therefore in this situation the E field in a direction tangent to the ring and the magnetic B field in circles about that ring combine to give an energy flow in space from outside the secondary ring directed into the outside of the ring representing the energy of Joule heating in the ring much as Feynman surmised for an ordinary DC current. Note that since the electrokinetic E field and the induced current in the ring change direction together, the energy flow is *always* into the ring regardless of the direction of the current or whether it is increasing or decreasing.

In the case of a transmission line carrying a higher frequency sinusoidal current, one can easily see that the voltage between say two parallel wires or a central wire and a shield in a coaxial cable impressed as a driving source on the line creates a radial electric field between the two conductors which in essence form a capacitor. That electric field combined with the circles of magnetic field about the conductors creates a cross product showing an energy flow down the line. In this case it is clear that not only is energy stored in space about the transmission line, but also that the energy down the line is carried in that space rather than by the currents in the conductors as might be thought. This action finds further verification in the fact that it's possible to guide electromagnetic energy with dielectric structures as well as conductive ones showing that currents in metals are not necessary for energy to flow down a guided structure.

Parallel Plate Capacitor with Current-Carrying wire in the Center

Consider now a charged parallel plate capacitor with circular plates of area, A, separated by a distance, D, bearing charges on the plates + q and – q leading to a potential difference of V where q = cV where c is defined as the capacitance of the apparatus. E = V/d gives the electric field between the plates and the capacitance can be shown to be only a function of geometry and is:

$$c = \frac{q}{V} = \frac{\varepsilon_o A}{d}$$

and the energy due to the electric field is pretty much concentrated between the two plates if we ignore fringing effects at the edges. When there is no magnetic field the energy remains trapped there. To discharge the capacitor we must move charge from one plate to the other and all moving charge constitutes a current that produces a magnetic field. That current and the magnetic field it produces will then depend upon just how we move those charges. We are accepting a circuit approximation here where all fields are concentrated close to devices and the proximity to other devices does not matter.
But before we consider that, let us examine a different apparatus as shown in Fig 2. Here we have our charged capacitor except that we cut a small hole in the top plate and run a wire vertically down to the grounded lower plate. We put a current in that wire equal to i. In this case we can now calculate the energy stored between the plates of the capacitor due to both the electric and magnetic fields. An arbitrary volume is constructed just inside the rim of the capacitor from plate to plate. Examining the directions of the fields, we see that the Poynting vector points everywhere toward the center wire of the volume

just as in the case of the wire above the Poynting integral shows a power equal to iV directed toward the center of the capacitor through any cylindrical surface inside the rim of the capacitor plates centered on the wire.

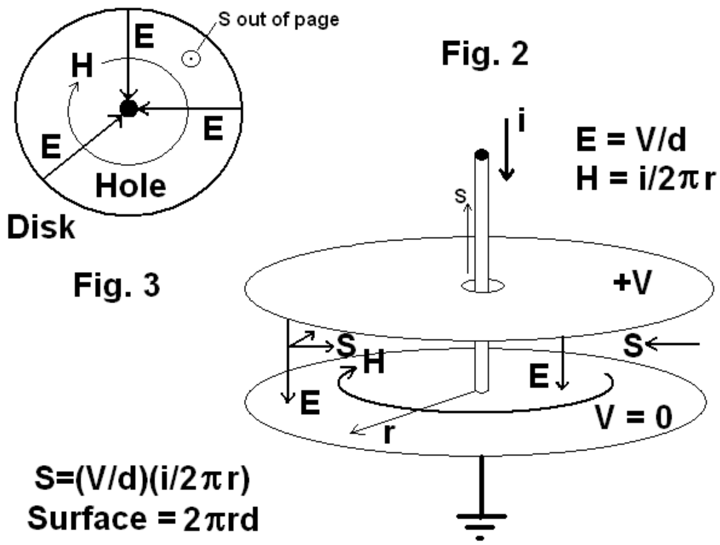

***Fig. 2, Fig 3, Poynting Apparatus*.**

This is a problem because both fields are not changing which means that the energy stored in the volume between the plates is not changing with time so the Poynting integral must be zero. We have missed something here! Then we remember the hole where the wire enters the capacitor as shown in Fig 3. The potential of the top plate is V while the potential of the current carrying wire is zero because it is highly conductive and attached to the lower plate. This means that there is a radial E field in the hole from the plate to the wire. In this case H is the same magnetic field as before and if we feed the current into our capacitor with a coaxial wire, we know that the Electric field inside the coax is given by :

$$E_r = \frac{V}{r \log\left(\frac{b}{a}\right)}$$

Where "a" is the diameter of our wire and "b" is the diameter of the "hole".

And if we cross that **E** field into the magnetic field from the wire we discover than this Poynting vector points up and outwardly from the top of our volume. Thus we have two areas of surface integration: The sides of the volume where the S vector points into the volume and the hole at the top where the S vector points out of the volume. This then gives us reason to believe that the two integrations can cancel (given no contribution from the rest of the area of the plates) and the Poynting integral can be zero which corresponds to our condition of a constant value of stored electromagnetic energy within that volume of space.

But we have a serious interpretation problem with the Poynting Vector as a power density in space. That interpretation says that we have a constant flow of energy in from the sides of the capacitor that flows across the volume and then squirts in a fountain out the hole in the top plate and does so for as long as we supply current to the wire! Note that since the wire has next to no resistance it consumes virtually no energy to operate. Hence we have invented a perpetual energy flow machine so we know our theory is in trouble! Furthermore, we can place the power supply providing current in the wire at a considerable distance where we cannot say that the fields from this apparatus are supplying the energy that is clearly said to be flowing into the sides of our capacitor and then out the hole in the top as a fountain into space again! If we use a coaxial cable to connect our apparatus to the power supply, the interpretation of S as a power flow leads us to conclude that we have a current flowing down the center wire and with a power flowing back up the space between the center wire and shield that is not only in the opposite direction but also is a function of the static voltage to which the capacitor is charged! And even more telling, since the wire is an excellent conductor it takes virtually no energy from the supply to provide the current for the system yet there is Vi power supposedly flowing back up the space in the coaxial current feed! The Poynting theorem is clearly correct, but its interpretation is clearly a mystery.

Constant Voltage Capacitor with Resistor in Center

So lets change the apparatus. We will replace the current carrying wire with a cylindrical resistor connected between the plates and connect a power supply to the capacitor to keep it at a constant voltage V. The current down the center resistor is now i = V/K which again gives rise to a circular magnetic field within the capacitor and the hole in the top is now closed. (K is used for resistance here to avoid confusion with the radii of the various parts.) This situation is identical to our situation above except that there is no "hole' for the energy to escape and all energy coming through our Poynting surface must be going into Joulean heating in the resistor bearing the current. Outside the resistor the Poynting vector integral is constant and equal to Vi outside the resistor and inside the capacitor and pointing inwardly on the surface of any radius right cylinder centered on the resistor that we imagine inside the top and bottom plates of the capacitor. Once again the value of the integral of the Poynting vector over that surface (irregardless of diameter of the cylinder so long as it is outside the resistor and inside the extent of the top and bottom plates) is equal to Vi where this time the current, i, is equal to V/K where K is the resistance of the resistor (to avoid confusion with r the radius of the imaginary cylinder, and R the radius of the resistor used to calculate the magnetic fields from the equations given above.).

Once again the Poynting theorem is correct in that the power calculated by the surface integral of the Poynting vector since the fields are still static and thus the field term is zero, is equal to the power being lost by the Joulean in the resistor which is obviously given by the product of the current through it and the voltage across it or iV. Hence those who accept the power density at a point thesis explain that we have energy from our power supply that provides the current and voltage to our apparatus, somehow radiates energy into space (and no matter how far it is away from our device) that flows around and enters into the side of the capacitor, flows across the volume in there (without changing the amount of stored energy in the space) and soaks into the resistor heating it. That is the philosophy of it.

But the philosophy that we use to calculate the Joulean heating integral in the Poynting theorem is quite different. There we multiply the conductance of the material with the **E** field = V/d and integrate over the volume to find the energy heating the material. We note that the conductance is zero outside the material so that this integral is *also* zero in the space between the capacitor plates where there is no resistive matter. Hence the **E** field created between the capacitor plates is irrelevant to the energy that goes into heating the material from this point of view. This is in stark contrast to the idea that energy enters the outside of the capacitor and somehow concentrates itself as it travels radially toward the central resistive element. Once again since electric and magnetic fields do not vary in time in the region between the capacitor plates it is clear that the amount of energy stored by them is not changing and hence those terms in the Poynting theorem are zero.

In general the Joulean heating integral is pretty much in line with the philosophy usually applied in the case of resistive heating. Suppose there is a battery that generates a potential. When you touch a wire to say the + terminal, the free - charge in that conductor is attracted to the + charge on that terminal. The free - charge sloshes in the wire toward the battery leaving the other end + at the battery essentially transferring that potential to the resistor at the other end. Note that all charge transporting this potential remains INSIDE the wire and all the fields outside the wire are due to the motion of that charge. The total wire material is charge neutral having large, but equal amounts of + and - charge. Charge from the battery can move onto the wire raising it's potential, but it must reside on the outside of the wire!

In our apparatus there is clearly an electric field inside the material of the resistor. This electric field accelerates electrons upward from the bottom of the resistor material. Each electron experiences a force F = qE which in turn gives it an acceleration F = ma which in turn (because electrons are very small and light) quickly accelerates them to a very high speed. But they cannot move very far and quickly smash into the atoms that make up the material. This imparts energy to the atoms that results in vibration which science has known for a long time is the essence of heat. All this atomic bombardment creates the electrical heating of the resistor. And as we know, accelerated charge and vibrating atoms give off radiation that is the energy lost through heating the resistor.

Since the charge in the resistor is drifting from one end to the other there is a current and that current creates a magnetic field. This magnetic field is in circles about the axis of the resistor and creates a Lorentz force on the moving electrons that tends to bend the electron trajectories toward the centerline of the resistor.

All this action is INSIDE the resistor. The electric field accelerates no charges outside the resistor because it's an insulator with no free charges to move. All the heating is the enhanced vibration of the material from free charges being accelerated and colliding with each other and the atomic matrix of the resistor material. Simple. No "nuts" theories of "rays" from outer space. No energy soaking in from the outside of the conductors. This is just a reasonable explanation of the motion of charged particles acting under well-known forces producing well-known heating.

Therefore once again as is typical of Maxwellian Electromagnetics we have a redundancy in ways to calculate phenomena both giving identical correct answers, but differing widely in philosophical implications. The existence of this redundancy with various interpretations was pointed out by Jefimenko[35] who first calculated the force between two dielectric plates four ways. He used the Lorentz equation, the scalar potential, the electric vector potential and finally the Maxwell stress equation. Then he did the same thing for the force acting between two current-carrying wires. He again uses the Lorentz equation, the magnetic vector potential, the magnetic scalar potential and finally the Maxwell stress integral. All these diverse methods give the "correct" value for the force in question. But the philosophy varies widely from operating on the various charge and current elements up to acting on an imaginary plane between the two plates or wires. It is not possible for all these interpretations to be correct. The Jefimenko calculations will be examined in greater detail later.

Before we leave this case, let us examine one more calculation. Let there be the same capacitor, central resistor being heated and a battery providing the energy to heat the resistor. It is important to note that the steady production of current and hence heat implies that non-conservative fields are involved as an electrostatic field is not capable of setting up steady currents.[36] Furthermore, since we are assuming circuit approximations all the fields are in close proximity to the elements (capacitor, battery, wires, etc.).

Therefore, if we now construct a large spherical surface surrounding the apparatus but at a distance where all fields are negligibly small, we have a situation where the integral of the Poynting vector over that surface is zero. The rate of change of the stored field energy term is also zero because no fields are changing with time. And finally we indeed do have a value for the Joule heating term due to the integral of the field within and the conductivity of the resistor. Hence our Poynting theorem becomes:

$$0 = -0 - Vi$$

Clearly, there is a problem here! And that problem is obviously that there is an additional term missing from the Poynting theorem! Just as Joule heating is not counted with energy transports out of the volume but is just considered a "loss", so a non-conservative battery, needs to be considered a "source" without consideration of the details of such energy. For this reason the "true" Poynting equation including a battery of voltage V supplying a current, i, actually is:

$$0 = -0 - Vi + Vi$$

This is now seen to be correct!

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[35] Jefimenko, Oleg D., , "Electromagnetic Retardation and the Theory of Relativity", Electret Scientific, 2004, Appendix 3, p. 302 ff.

[36] Plonsey and Colin, "Principles and Applications of Electromagnetic Fields", Mc Graw-Hill, NY, Section 5.2 p. 167 ff.

A few other calculations are instructive. Instead of cylindrical volumes centered on the resistor axis for which we found the surface integral of the pointing vector to always be equal to Vi no matter what the diameter so long as you stayed outside the resistor and inside the perimeter of the capacitor, we will just take a pie-shaped fraction of the cylindrical volume as our surface of integration as shown in Fig. 4.

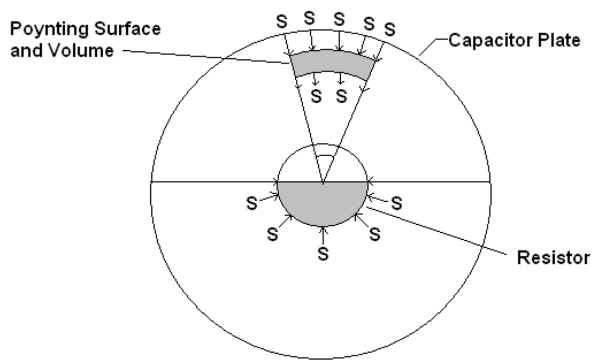

Fig. 4. Additional Poynting Surface/Volumes within the Capacitor/Resistor Apparatus.

Note that the Poynting vector is perpendicular to the surface normal on the top and bottom of the volume as well as on the sides. Hence, all contributions come from the cylindrical surfaces. But since the surface integral of any cylinder is equal to Vi, the values of the inner and outer surfaces are equal, but opposite in sign (because the surface normal changes direction) and hence cancel. From this we can conclude that any such volume that does not include the resistor produces a Poynting integral of zero. Furthermore since in this region all fields are static and conductivity is zero, we again arrive at a null Poynting theorem equation. Yet there is the same suggesting of an energy flow through the volume just as in the case of the DC wire.

If the volume is expanded by shrinking the inner cylinder until it coincides with resistor surface the Poyting equation is still null, but as we move the inner surface inside the resistor, we must remember that the magnetic field begins to fall for $r < R$ where R is the radius of the cylindrical resistor. Thus, when this happens the inner and outer surface no longer cancel and the Poynting surface integral assumes a finite value and that value will be equal to the amount of power dissipated in the volume of the resistor included in the surface.

If the pie shape is expanded to include half the cylinder and the Poynting surface is just outside the resistor and then slices it in half. The value of the Poynting surface integral is clearly ½ Vi and since the normal is perpendicular to the Poynting vector on the flat side, top and bottom this gives the total value. On the other hand since the volume of the resistor giving off heat is ½ of the total it's clear that the power going into heat is ½ Vi as well. And the same arguments hold for any fraction of the total cylinder. Thus, the consideration of the Poynting theorem in revealing power flows is clearly valid, while the interpretation of S as a power density seems suspect in the case of DC fields.

A final case to consider would be that of totally conservative fields which would be the same apparatus, but without the battery. In that case there is only a capacitor with a certain amount of energy stored between it's plates in an electric field, which is slowly dumped into the central resistor over time.

Capacitor Discharging by a Resistor

To complete this discussion of Poynting energy flow, let's consider the simple case of the capacitor in question but with no power supply to maintain the voltage. In this case there are no non-conservative fields (battery etc.) and because it's all conservative the flow of current cannot be maintained and all fields must eventually drop to zero implying that all energy stored in the capacitor is moved into the resistor becoming heat.

If the constant voltage battery in the example above is disconnected we know that there will be energy stored in the electric field of the capacitor and that energy will flow as power into the resistor heating it until the voltage and current finally is zero. We can help get a handle on the situation by using the circuit approximations we know to be generally valid in this case..

There is an initial voltage on the capacitor, V_o, which also appears on the resistor at the same initial instant, t = 0. As energy stored in the capacitor flows into the resistor it heats it, which appears in our representation as an energy loss.
In circuit theory we know that the power flowing into the resistor is given by:

$$W = i(t)V(t) = [i(t)]^2 R = \frac{[V(t)]^2}{R}$$

And Voltage and current are related everywhere in the circuit by:

$$V(t) = I(t)R$$

And that the energy stored in the capacitor at any time, t, is given by:

$$U = \frac{1}{2}C[V(t)]^2$$

Which means that the power flow out of the capacitor is given by:

$$W = \frac{d}{dt}\left[\frac{1}{2}C[V(t)]^2\right]$$

But for this apparatus we actually know the time functions of the voltage and current:

$$V(t) = V_o e^{\frac{-t}{RC}}$$

$$i(t) = \frac{V_o}{R} e^{\frac{-t}{RC}}$$

Where we are using the typical circuit designation of R for resistance. Thus the power flowing into R at some time t is given by:

$$W = i(t)V(t) = V_o e^{\frac{-t}{RC}} \frac{V_o}{R} e^{\frac{-t}{RC}} = \frac{V_o^2}{R} e^{\frac{-2t}{RC}}$$

Whereas the power flow out of the capacitor is given by:

$$W = \frac{1}{2}\frac{d}{dt}\left[C[V(t)]^2\right] = \frac{C}{2}\frac{d}{dt}\left[V_o^2 e^{\frac{-2t}{RC}}\right] = \frac{CV_o^2}{2}\left(\frac{-2}{RC}\right)e^{\frac{-2t}{RC}} = -\frac{V_o^2}{R} e^{\frac{-2t}{RC}}$$

Which is a clear demonstration that the energy stored in the electric field of the capacitor is going into the resistor where it becomes heat and that power flow at any instant is equal and opposite in the two circuit elements.

However, out interest here is in the Poynting theorem and the assumption that the Poynting vector is a power density.

Since we already know that the Joulean heating in the resistor is due solely to the energy stored in the electric field of the capacitor, we can surmise that the surface integral of the Poynting vector over a volume enclosing the device must be zero. It is clear by our circuit approximation, which demands that fields be negligible at distances from the circuit components that if we enclose the capacitor and resistor in a large sphere the E and H fields will be negligible on that surface and the Poynting theorem once again yields a correct result.

But previously we found for a capacitor with a central resistor that the Poynting integral is equal to iV for any cylinder enclosing the system. Thus, we now have THREE terms all equal to the same power flow, which form an equation, which is can't possibly correct! So clearly we have done something wrong.

The thing that is wrong is that our previous calculations took advantage of the fact that we had current flowing in a long wire to create the H field. Here the current is flowing in a short stubby wire so our previous calculations do not apply. In fact, in general we know that as we move from a long wire where the magnetic field falls off as 1/r to a short wire, where at a distance the field will actually fall off as $1/r^{2,}$ there will also be serious "fringing" throughout the capacitor. This totally invalidates our previous approach.

So it is necessary to change our apparatus. We can do this by placing the capacitor in the canter of one long wire and the resistor in the center of another. We can complete the circuit by connecting the wires at the ends at some distance. Having done this let us calculate the Poynting theorem and Poyting vectors separately for the resistor and the capacitor.

At the capacitor, the Poynting theorem stored energy power flow term will be given by:

$$W = -\frac{1}{2}\frac{d}{dt}\int_{volume}\vec{E}\times\vec{D}\,dv = -\frac{\varepsilon_o}{2}\frac{d}{dt}\int_{volume}[E(t)]^2\,dv = -\frac{\varepsilon_o}{2}\frac{d}{dt}\frac{[V(t)]^2}{d^2}\int_{volume}dv = -\frac{\varepsilon_o}{2}Ad\frac{d}{dt}\frac{[V(t)]^2}{d^2}$$

Where A is the area of the capacitor plate and d is the plate separation. For such a parallel plate capacitor the capacitance, C, is given by:

$$C = \frac{\varepsilon_o A}{d}$$

This:

$$W = -\frac{-\varepsilon_o A}{2d}\frac{d}{dt}\left[V_o^2 e^{\frac{-2t}{RC}}\right] = -\frac{\varepsilon_o A}{2d}\frac{-2V_o^2}{RC}e^{\frac{-2t}{RC}} = +\frac{V_o^2}{R}e^{\frac{-2t}{RC}}$$

Which is the correct value for power flow from the capacitor.

Since the capacitor is now in the center of a long wire we can calculate the Poynting integral as before. One might ask how we can ignore the gap in the center of the capacitor where there is no wire. The traditional explanation is that in this region there is a “displacement current” which were it inside a black box would be indistinguishable from the conduction current in a wire.[37] The only problem with this is that the existence of convection currents is in doubt as none has ever been reliably measured. Calculations have been done on parallel plate capacitors that show that what appears to be measured as a displacement current can be explained by the currents flowing on the plates of the capacitor as the amount of charge changes. In any event, these arguments allow the gap in the wire to be assumed to act as if there were conduction current there and this allows the calculation of the magnetic field within the capacitor. Thus, as above, we find that the Poynting integral over the surface of the volume of the interior of the capacitor is:

$$W = \int \vec{S}\bullet\vec{u}\,ds = \vec{E}\times\vec{H}\,(\pi 2rd) = \frac{V(t)}{d}\frac{i(t)}{\pi 2r}(\pi 2rd) = \frac{[V(t)]^2}{R} = +\frac{V_o^2}{R}e^{\frac{-2t}{RC}}$$

Which is exactly equal to the power flow from the stored E field and since there are no materials for Joulean loss within the volume, the Poynting theorem once again gives a correct answer. However, it is important to notice that now the power flow at the capacitor as interpreted by the Poynting vector is no longer into the capacitor but instead is allegedly spraying power out around the edges of the gap in the device.

[37] Plonsey and Colin, “Principles and Applications of Electromagnetic Fields”, Mc Graw-Hill, NY, Section 5.9 p. 188-189.

Similarly the Poynting theorem can be calculated for a volume that just encloses the resistor located in the middle of a long straight wire located at some distance from the capacitor.

In this case the Poynting theorem Joulean power loss is given by:

$$W = -\int \sigma [E(t)]^2 dv = -\sigma [E(t)]^2 \pi B^2 l = -\sigma \pi B^2 l \frac{[V(t)]^2}{l^2}$$

Where B is the radius of the cylindrical resistor and l is it's length. But in the resistor the resistance of it is given by:

$$R = \frac{l}{\sigma \pi B^2}$$

Thus

$$W = -\frac{[V(t)]^2}{R} = -\frac{V_o^2}{R} e^{\frac{-2t}{RC}}$$

Which allows us to then calculate the Poynting vector surface integral:

$$W = \int \vec{S} \bullet \vec{u}\, ds = -\int \frac{V(t)}{l} \frac{I(t)}{2\pi B} ds = -V(t) i(t) = -\frac{V_o^2}{R} e^{\frac{-2t}{RC}}$$

Which leaves the question of the energy flow into electric and magnetic fields within the resistor. It is clear that there is energy from the capacitor that creates a current through the resistor that creates an H field and a voltage across it that implies an E field. These must also store energy also even though it is obvious that this energy must all be converted to heat by the time the current and voltage goes to zero.

Notice that the fields immediately reach their maximum values (and hence maximum stored energy) once the capacitor is connected to the resistor. There is obviously a "pulse" of energy from that stored in the capacitor that immediately goes into the electric and magnetic fields of the resistor. From that point on the stored energy slowly "leaks" into the resistor as heat as the voltage and current drop. Resistors are not energy storage devices so that the energy stored within them is quite small reflected in their tiny values of capacitance and inductance. For this reason it is clear that the field energy term with regard to the resistor is negligible so that the major energy values in the Poynting theorem are the Poynting integral and Joulean heating terms. As we see above.

Circuit theory

The useful assumption used in circuit theory is that not only are electric and magnetic fields contained in spaces very close to the circuit element, but also the elements are idealized such that capacitors are said to only have stored electric energy, inductors only magnetic field energy and resistors no stored energy at all.

It is clear from the above examples that within a resistor there are both magnetic and electric fields that *must* be storing energy when currents are present. The same goes for the magnetic fields in a capacitor or electric fields about inductors. Generally speaking in circuit theory these minor amounts of stored energy are ignored so as to "idealize" the circuit element. In more detailed calculations these energies are added to the circuit model as discrete components, such as a capacitor or inductor to represent the stored electric and magnetic energies within a resistor.

From all this we can see that in doing these kinds of calculations one must be very careful of models, approximations and assumptions if spurious results are to be avoided. Furthermore notice that in this case the interpretation of the Poynting vector as a power density leads to an unexplainable situation where power is supposedly spewing out of the gap of the capacitor into space where it travels across any distance we choose and then concentrates around the outside of the resistor pouring into the material to heat it.

There is little to support such a philosophy.

The Poynting Philosophy

Now having examined a number of cases where there is apparent energy transport that is reflected in a Poynting calculation, we need to step back and ask "what does it all mean?" You will recall the musings of Maxwell who noted that energy is able to be transported only two ways: Kinetically by the physical motion and by wave propagation in a medium. Later when we examine the Jefimenko equations for E and H fields we will observe that electromagnetic radiation is generated by the time rate of change of currents, which is to say the rate of change of the velocity of charge, which is to say accelerated charges. Such currents provide a source from which both E and H fields travel out into space through the medium supporting them (without our actually saying what that medium actually is) leaving the sources behind. Hence in this case the crossed E and H fields seem to represent stresses in the medium, which is to say the transport of energy.

In the case of Joule heating, on the other hand, the energy appears to be largely kinetic. Where there is no acceleration of the source charge (DC) and there no radiation terms in Jefimenko's equations. The heating of a resistor is widely assumed due to the kinetic energy of moving charges colliding with each other and atoms in the material. What our examples of the Poynting theorem above seem to show is that while there may be no changing of the energy stored in space, there is nonetheless an energy transport reflected in the Poynting theorem which correctly calculates the transport of that energy from here to there such as from one end of a wire to another into a resistor.

Thus in the kinetic charge transport case we know that when there is a current in space, there is both charge which is creating an electric field and charge motion which is creating a magnetic field. Hence even though the actually energy may be kinetic, these attached fields clearly represent the *mechanisms and motions* of the kinetic charges that may be carrying the energy. However, is appears to be a stretch to assume that when these fields are related to kinetic energy transport that the actual energy is transported "in the fields".

As we've pointed out many times before, fields are not things. They are mathematical representations of phenomena in space related to its fundamental structure. Hence while space may carry energy both as waves in a medium or as motion of particles (note that moving uncharged particles such as neutrons also transport energy, but produce no Poynting values because they have no electric or magnetic fields) the *field* isn't some objective thing, but merely a way of describing how the fundamental properties of space operate. This is important to remember because as of today, science still has no idea what charge is, what space is "made of", or how the electric or magnetic forces between objects that field theory describes are produced. We *describe* them, but we don't *explain them!*

Our philosophical bottom line here is that there seems to be two distinct ways of looking at the Poynting vector and the Poynting theorem. In one case when we are out in space far removed from the sources of electromagnetic radiation, we find that calculating the Poynting vector from electric and magnetic fields in space actually produces a representation of how energy is distributed by waves propagating through empty space. Hence the Poynting vector not only represents the flow of energy or power at a point, but we shall se that it also gives a representation of a flow of momentum in space carried with that energy flow.

On the other hand, when energy is carried kinetically by the motion of charges as in the case of static unchanging E and H fields as often occur in the transmission of power by direct currents by the Poynting theorem the fields produced by that kinetic transmission of energy when averaged are still capable of redundantly revealing the values of the energy being transported, but the interpretation of the Poynting vector itself as representing energy transport density at a point in space seems very much in doubt. In our examples above we see there is little experimental justification for assuming that energy in our central resistor is pouring in through the capacitor from space. Yet, as we have seen the averages represented by the Poynting theorem remain valid correctly giving the values of resistor heating and the like.

It is this difference in philosophy between these two cases that has led many including Feynman above, to conclude that we don't know where energy is stored in space. Yet when it comes to the storage of energy in space by a single E or H field, Maxwell's stress theory seems to be a reasonable philosophy. That implies that the value of these fields squared is indeed representative of how much energy is stored at a point in space. And furthermore when both fields are present, and thus a Poynting vector exists, this seems a necessary condition for the transport of that stored energy from one area of space to another.

But as we saw in the case of the discharging capacitor above, one must ask whether or not the Poynting vector represents the flow of actual energy at a point in space or whether it is merely the average of those fields that will give the energy transport by the motion of charges at some distance from the fields. In other words, is the energy stored in the space between the plates in the capacitor discharging into the central resistor moving sideways into the resistor or are the charges on the capacitor plates moving radially to the center where they flow through the resistor heating it and neutralizing the potential difference between the two plates. The later viewpoint interprets this experiment as a kinetic one with energy transported by the movement of charges across the capacitor plates and through the resistor. A Poynting philosophy that totally ignores the kinetic motion of the charges producing the fields in this case seems hardly justified. On the other hand it is important to understand that each charge stores electric energy in space and when it moves it stores magnetic energy in space, but when two opposite charges come to rest near each other as in an atom of a charge-neutral material both those energies stored in space are forced to come out of space as the fields go to zero.

There is much to think about here.

Our further discussion here, however, will center upon discussions of the *direction* of power flow rather than it's magnitude. In particular we shall be interested in examining power flows in and out of space in regard to the various terms in our previous causal expressions for E and H fields in common simplified situations.

Microscopic Fields

Observing the above forms of the Poynting vector we note that we can define it as:

$$\vec{S} = \frac{1}{\mu_o}\vec{E} \times \vec{B}$$

Our previous definition can be used only in free space where $\mu = \mu_o$ and $\varepsilon = \varepsilon_o$ or where the relations hold:

$$\vec{D} = \varepsilon\ \vec{E} \quad and \quad \vec{B} = \mu\ \vec{H}$$

Where ε and μ are constants, which limits usage to cases where the medium is linear, non-dispersive and uniform (isotropic). In a vacuum the "E x H" and "E x B" versions of the Poynting vector are equivalent. In this microscopic case the "bound currents" in magnetization per unit volume is not included in the magnetic field and considered separately. Hence, the term "microscopic fields" is used.

Our interest here, however, is more general and will be restricted to fields in a vacuum primarily to demonstrate energy flows due to electromagnetics in space. A more detailed discussion of microscopic field is given in the referenced paper.[38] Energy flows in materials is a whole other complex topic.

[38] Richter, F.; Florian, M.; Henneberger, K. (2008). "Poynting's theorem and energy conservation in the propagation of light in bounded media". *Europhys. Lett.* **81** (6): 67005.; See http://iopscience.iop.org/0295-5075/81/6/67005/

Maxwell Electromagnetic Theory

To briefly review the basic premise of electromagnetics, we find that since space has various properties, Einstein has noted that therefore space is not "empty" but must consist of something possessing those properties. The idea of behavior presupposes the existence of something doing the behaving. For lack of a better term we shall call this substance of space that acts as a transmission medium for waves "aether" following the traditional term used by Maxwell, though we have noted that our modern view of this medium must be quite different from the 19^{th} century ideas common in his day.

At this point simply noting the physical nature of waves and their propagation we observe that energy is transmitted though media by creating stresses in it which through interconnections within the substance of the medium are transmitted from one location to another. The resultant "stresses" typically obey solutions to wave equations that are the mathematical model for this energy propagation through space. Maxwell noted that no other means of energy transmission over distance is possible except by the kinetic energy of projectiles. Only these two methods of communication are known to exist. Current theories of physics that deny any electromagnetic medium and propose the transmission of energy by essentially "magic" are less than satisfying.

Maxwell then proposed that these stresses induced in the medium therefore represent stored energy and that electric and magnetic fields are mathematical models representing that energy stored in space. On that basis he derives relationships that relate the measured values of electric and magnetic fields in space to the amount of energy stored with in a given volume of space. In the case where material objects exist in that volume, resistive Joule losses can also be included in the equation. But these losses are tricky.

Simultaneous E and H Fields

Maxwell suggested that static fields, E and H, represent stored energy in space. Total energy is the sum of those two stored energies. But how does energy get in and out of space? Can only a single field create momentum in space? **E** and **H** fields have very different characters. An electrostatic E field, for example, is produced by charge. If the object is small the field is radial out into space. This kind of field is called an *irrotational field*. It is not hard to imagine this field representing the stresses in a spring-like medium produced by whatever "charge" happens to be. Therefore, electric fields appear to be some kind of "pressure" applied to space, which creates a stress in the medium of space. That stress travels radially outward at no more than the speed of light. By geometry alone we can see that amount of stress per unit volume of space becomes less and less as the stress spreads into greater and greater volumes of space.

One can imagine a model where say positive charges create a positive pressure into space, or a "source" stressing the "springs" of space and propagating the stresses through it. This pressure gives a basis for the force on charges. Obviously if we have two sources of positive pressure located near each other they will tend to push each other apart. If we model negative charge also as a pressure but now a reduction of the normal pressure of space or a "sink". Should there be both a positive and negative electric field in a region of space the pressures cancel any stress induced into the medium and no energy is stored in the "springs" of space. The model doesn't exactly work to explain the repulsion of negative charges, but these kinds of "source" and "sink" models are commonly used in textbooks to illustrate static irrotational electric fields.[39]

A magnetic field, on the other hand, is quite different. Magnetic fields as far as is known are all produced from currents. Currents are a flow of charges. In this case the field is formed of closed circles. These circles have the path of the moving charge as their origin and hence any moving charge produces circles of magnetic field around its path of motion as the axis. This field falls off as the distance increases and also varies with the trigonometric sin of the angle to the path being maximum at right angles to the motion and zero in the direction of motion. Hence a moving charge produces no magnetic field in the direction of motion or in the direction from which it came. This kind of field is called a *solenoidal field*.

This completely different situation leads us to think about "stresses in space" in a different way. Because the field is solenoidal, we suggest a model whereby a magnetic field could be thought of as a superconducting circulation in space about the path of the charge. That circulation thus hints at the nature of the magnetic energy stored in space. Think of it as "charge" as consisting of some kind of "spin". When a charged particle is accelerated and an electric field is applied. That field can be thought of as flipping over the randomly oriented spins of electrons into the direction of the E field. That in turn creates an array of charges with spins aligned that can act as tiny "paddle wheels" in the aether inducing the circulation in space about the moving charges.[40] Mathematically any general field can be created out of a combination of *irrotational* and *solenoidal* fields.

The above simple models are not presented as an actual description of reality or even a mathematical model of electromagnetics. We think Maxwell's equations are quite sufficient in that regard. But rather they are suggested as "thinking tools" to help understand the basic interactions of energy stored in space. For example the "circulation" of a magnetic field in space can also be imagined as the winding of a large "clock spring" given that wave propagation implies a connection between one bit of space and the next.

Looking at the causal equations for an E field we find that electrostatic E fields arise from charge distributions. If we have a distribution of charges in space, we have an E field about those charges extending to "infinity"[41]. And that field represents stored energy. But how did that that field get *into* space? Clearly that charge had to be *created* in that

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[39] Halliday and Resnick, "Physics for Students of Science & Eng..", John Wiley, 1960, Vol. I pp. 386-387.

[40] See for example Plonsey and Colin op cit. Pp222-223 for discussion of curl as "tiny paddle wheels:" and also see Maxwell, : Theory of Molecular Vortices applies to Electric Currents" , Phil. Mag., Ser 4,Vol 21, #139, March 1861, pp338-348 for his "ball bearing" theory of curl. Best theory is that of Airy on water waves extrapolated to extra-dimensional surface waves on the aether of 3D space.

[41] Actually since the E field travels no faster than the speed of light, any field from a given configuration never reaches "infinity" in a finite amount of time. Hence really only a retarded "far field" makes any practical sense.

location. But by conservation of charge we know that charge cannot be created or destroyed. Therefore, to produce a time rate of change of charge density that will create a changing E field in space, it is obvious that charge must be carried to that distribution or carried away from it. This fact is expressed in what is called the *continuity equation.*

$$\nabla \cdot \vec{J} = -\frac{\partial \rho}{\partial t}$$

Which shows the obvious fact that since charge cannot be created or destroyed, to change a charge density a flow of charge is needed and that flow constitutes an electric current. Therefore, any creation or destruction of an Electrostatic E field demands a simultaneous current, which immediately implies a simultaneous magnetic field so an electrostatic E field cannot be changed in magnitude without the creation of a temporary magnetic field even though that magnetic field will drop to zero and disappear once the flow of charge (current) stops. This fits exactly with the Poynting equation as a vector being the cross product of an electric and magnetic field. Both **E** and **H** are needed if an energy flow is to occur.

Stored Energy Flow about a Moving Charge

When a point charge sits in empty space, it produces an electrostatic irrotational field that represents energy stored in space as “stresses” about that charge. Think of it as a concrete block sitting on bedsprings. As soon as we try to move that charge to a different location several things happen. The first is that as the charge moves the electric field distribution moves with that charge following it exactly (except for retardation effects as those field changes propagate out from the moving charge at the speed of light). But any moving charge constitutes a current and a current creates a Biot-Savart magnetic solenoidal field about the direction of motion. Note the magnetic field is zero along the direction of motion so there is no magnetic deflection of the path.

Therefore, while the charge is moving at constant velocity we have both an electrostatic E field and solenoidal magnetic field in space about that moving charge. This allows the calculation of the field of Poynting vectors in space about the moving charge, which gives the flow of energy in space. Figure 2. shows the nature of that Poynting field which clearly shows a flow of energy in space from the area behind the moving charge where it was previously stored, to in front of the moving charge where it is going to be stored. Thus, it is easily seen that for this reason a “gob” of energy follows a charge around as it is moved about in space. The very fact of motion creates a Poynting energy flow from where the energy was stored to where it will be stored. And note that since the Poynting vector is the product of electric and magnetic fields which by the causal Maxwell equations are seen to fall off as the inverse square of distance from the source elements, which means that the power flow falls as the inverse 4^{th} power of distance, the total energy “gob” and energy flows about moving charges tend to be very much concentrated close to the charge.

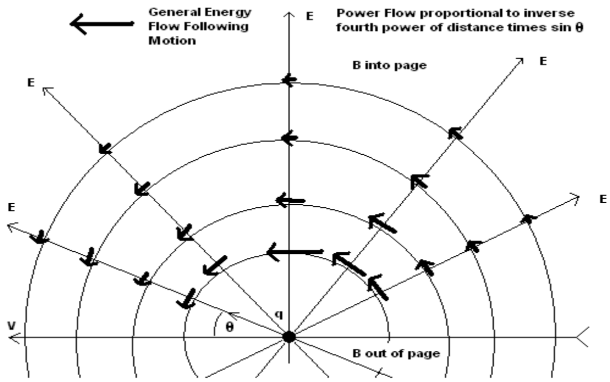

Figure 5. Energy Flow in Space about a charge moving at constant velocity.

Similarly we can examine Electrokinetic E fields. In this case it is seen that the source of *both* Electrokinetic E fields and magnetokinetic H fields is a changing current. Therefore, for any changing current, both these fields are created simultaneously. And from our causal equations we see that they are created at right angles to each other at any observation point since the electrokinetic field is in the direction of the current while the magnetokinetic fields results from the cross product of the current direction with a unit vector in the observation direction.

Above, we saw a charged particle moving at constant velocity creating a magnetic field. But then the questions arise of how the magnetic field was created and how the energy stored in that magnetic field got there. Given that the velocity of the charge was initially zero it had to be accelerated to the final velocity and that acceleration represents an increasing current. Therefore an electrokinetic E field is created about the charge in a direction parallel to it's velocity. As shall be seen below the combination of the Electrokinetic E field and the Biot-Savart B field generates a Poynting vector out into space from the moving charge that represents the flow of energy into space that is stored in the magnetic field aspect of space due to charge motion. Similarly when the particle decelerates and stops then energy flow for the magnetic field is back out of space because the magnetic field disappears. The electrostatic energy storage in space due to the charges, however, remains in all cases.

Thus, we see that while static electric and magnetic fields can exist in space, their creation or destruction demands simultaneous E and H fields that create a Poynting vector indicating energy and momentum flows in space.

Energy Flows from a Current in Space

Before this discussion begins we will reproduce here our former causal Jefimenko equations for an electric field E and a magnetic field H:

The Causal Equations for E and H

$$\vec{E} = \frac{1}{4\pi\varepsilon_o}\int\left(\frac{[\rho]}{r^2}+\frac{1}{rc}\frac{\partial[\rho]}{\partial t}\right)\vec{r_u}\,dv' - \frac{1}{4\pi\varepsilon_o c^2}\int\frac{1}{r}\left[\frac{\partial\vec{J}}{\partial t}\right]dv'$$

and

$$\vec{H} = \frac{1}{4\pi}\int\left(\frac{[\vec{J}]}{r^2}+\frac{1}{rc}\frac{\partial[\vec{J}]}{\partial t}\right) x\ \vec{r_u}\ dv'$$

If now there is considered a short piece of wire in space traveling at some constant velocity in an inertial laboratory frame bearing a constant current I, with the observer traveling at the same velocity in the same frame, we can note that since this wire is a metallic substance containing equal numbers of positive and negative charges the electrostatic terms involving charge in the equation for E are zero as an integral over all charges is zero. Furthermore, the two kinetic terms for both E and H are likewise zero because the current is constant and its derivative is therefore zero. This leaves us with a single term for the fields due to this current. That field is what we have termed the Biot-Savart magnetostatic field created by the constant current in the wire. The E fields are all zero.

From our previous considerations we know that this magnetic field in space about the wire represents stored energy assumed to be some kind of “stress” of space itself. Since all E fields are zero this insures that the Poynting vector, S = E x H is also zero everywhere in space about the wire. This fact assures us that there is no energy flow into or out of the space about the wire. So the situation is that this wire essentially has a large gob of energy around itself that moves with it without resistance anywhere the current-carrying wire goes. The only operative effect of interest would be retardation, which causes the fields to lag behind the moving wire at high speeds and changes the apparent shape of these fields to an observer in a stationary laboratory frame. Such a moving current system can be commonly seen in say a super-conducting MRI magnet where a constant current is established and the system then drags a large magnetic field around with it wherever it is moved with the attendant large amount of stored magnetic energy in the space around the current. When temperatures rise and superconductivity fails that stored energy is suddenly returned to the conductor as Joule heating as the stored energy forces current through the now non-superconducting wire.

If we ask how that situation got into space in the first place, we must start with a wire with no current and in some manner, such as a linear ramp function, raise the current in the wire to its constant value. If we do this we observe that the electrokinetic term for the E field suddenly jumps to a fixed non-zero value. This, therefore, means that the E field is no longer zero, which insures the Poynting vector is no longer zero which means an instantaneous power flow now exists in space.

We know that the electrokinetic E field occurs about the wire in a direction anti-parallel to the increasing current in the wire, which because of the minus sign in the term means that the electrokinetic E field about the wire is a vector pointing opposite to the current flow. If we cross this vector into the previously discussed Biot-Savart H field which occurs in circles about the current, we find that the direction of the Poynting vector is *away* from the wire at all points in space. The clear indication here is that the increasing current in the wire is causing an energy flow into the space surrounding the wire until the final value of current is reached when the electrokinetic E_K field falls to zero and the stored energy is “locked in”.

Similarly if once the current is established, we begin to linearly reduce the current value, again the derivative of the current attains a constant value, but this time the slope is negative which changes the sign of the Electrokinetic E field. For this reason the sign of the Poynting vector is also changed which therefore indicates a flow of energy *out* of the space surrounding the wire where it is stored and into the wire. In simple terms, the stored magnetic energy is returned from space. And this continues until the current is zero, which means there is not longer a magnetic field and hence no more energy stored in space. The situation is diagrammed in Figure 3.

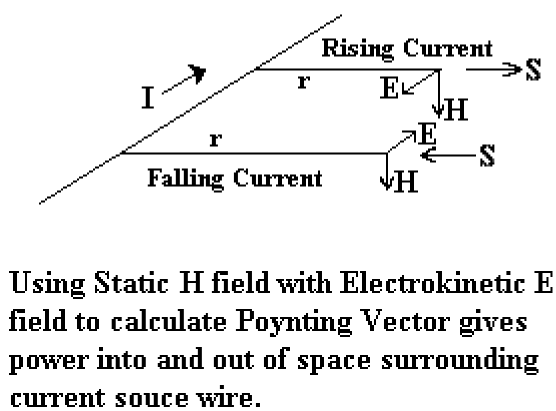

Figure 6. Energy flows into and out of space from current in a wire.

It should be noted that the negative sign in the electrokinetic field represents Lentz’s law. This means that if the current is increasing the electrokinetic field opposes that increase, or in other words is directed against the current, but when the current is decreasing the electrokinetic field is in the direction of the current attempting to sustain it. Since the direction of the magnetic field changes with the direction of the current, should the current flow in the opposite direction, both the E and H fields change direction leaving the direction of the energy flow the same. Hence an increasing current always flows energy into space and a decreasing current draws that energy back from space.

But the Biot-Savart field is not the only magnetic field in space about the wire. The changing currents have *also* given rise to an additional magnetic field, what we have termed the magnetokinetic field. This field also produces a Poynting vector indicating energy flows in space about the wire. In the initial case of a rising current in the wire, this additional magnetic field is positive and hence is in the same direction as the Biot-Savart field. Therefore identical to our previous consideration we find an additional energy flow out into space from the wire.

But the interesting thing about these kinetic fields is that because these fields depend upon the derivative of the current magnitude, both reverse when the current time rate of change changes direction, so when a rising current falls *both* the magnetokinetic field and the electric electrokinetic fields change sign, which keeps the energy flow outward into space! In other words this energy is *always* launched into space and never returns. Observe that the energy flow is also outward for current flowing in the opposite direction. Therefore, if our driving current were a sinusoid, it is clear that whether the current is positive or negative or rising or falling with an increasing or decreasing value and a positive or negative slope, the flow of energy is *always* outward into space to "infinity". Thus, it seems reasonable to identify this Poynting term with radiation.

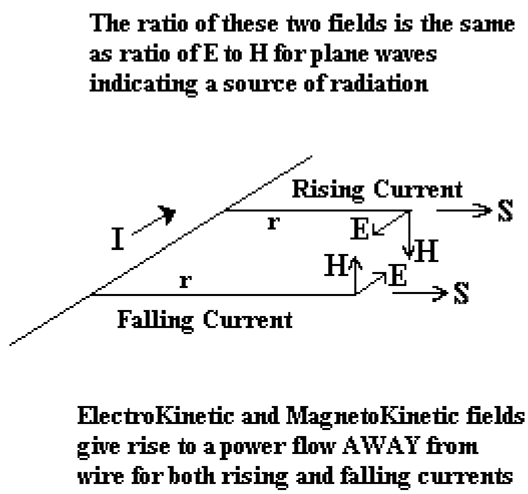

Figure 7. Outbound energy flow from kinetic fields.

It is important to remember that fields used in Poynting calculations are the *total* fields added together before the energy calculation is made. In the radiation case, what that means is that energy into space around a wire consists of the electrokinetic E field and the *total* magnetic field, which consists of *both* the Biot-Savart field and the magnetokinetic field. Since former falls off with distance from the current sources as $1/r^2$ while the magnetokinetic field falls off as 1/r, the situation shown in Figure 4., holds only at large distances where the total magnetic field is reversing in time. Close to the current source the Biot-Savart field dominates and the magnetic portion of the Poynting vector is merely modulated by the magnetokinetic field rather than reversed by it.

What this means is those fields close to the current source show it acts as a reactive inductive/capacitive device with energy flowing in and out of near space. Only at a distance where the magnetokinetic field begins dominate and cause the magnetic field to reverse does the constant outward flow of radiated energy take place.

Near Fields, Fresnel and Far Fields

When the subject of radiation and antenna is discussed there is always the question of near fields and far fields. Far fields as we've seen above represent the transmitted information that is the energy density at the receiver location. Near fields on the other hand occur near the antenna and represent energy stored in space and hence are reflected in the reactive part of the antenna driving impedance. Both issues are of interest.

Traditionally near and far field determinations are made by expanding the given solution of Maxwell's equations for the antenna in question in a series where the near and far fields are identified with the terms according to how rapidly fields fall off with distance and other criteria. The fields with the least rapid fall-off are termed far field. Often other regions are defined termed near field close to radiating elements and an intermediate zone termed the Fresnel region. Here we simplify this to only near and far field zones. The causal Maxwell equations discussed above gives us a very simple new approach.

We have identified two Poynting vectors of electromagnetic power flow. There is a reactive flow in and out of space through a product of the electrokinetic E_K field and the Biot-Savart H field. We have also identified a continuous radiation power flow out into space as a product of the Electrokinetic E_K field and the Magnetokinetic $H_{K.}$ The Poynting Vector representing a power flow in space is given by:

$$\vec{S} = \vec{E}_K \times \vec{H}$$

Therefore, at a given elementary volume of space dv we can simply multiply the causal coefficients for both the near field and the far field E and H vectors and arbitrarily define the transition between near and far field as being when the two power densities are equal, where we find:

$$\vec{E}_K \times \vec{H} = \vec{E}_K \times \vec{H}_K$$

$$\frac{1}{4\pi\varepsilon_o c^2 r}\frac{dJ}{dt} \times \frac{1}{4\pi r^2} J = \frac{1}{4\pi\varepsilon_o c^2 r}\frac{dJ}{dt} \times \frac{1}{4\pi r c}\frac{dJ}{dt}$$

Which simplifies quite readily to a transition distance of :

$$r = c\frac{J}{\frac{dJ}{dt}}$$

If we assume a sinusoidal excitation [J = sin(ωt)] as is normal in EM radiation and examine the amplitudes we find:

$$r = \frac{c}{\omega} = \frac{c}{2\pi f} = \frac{\lambda}{2\pi} \approx \frac{\lambda}{6}$$

Retarded Energy Flow from Charges in Space

Another situation could be a charged object located somewhere in space. Once again the charges on that object create an electrostatic E field about the object, which becomes spherically radial at large distances where all distances from the observer to points on the source are essentially the same. Like the Biot-Savart magnetic field this electrostatic field as a magnitude that falls off as the square of the distance from the source. The result is another case of a gob of energy in space surrounding a source, which in this case is charge. If that source moves, it again drags that gob of energy with it. This situation was previously examined in Figure 2., above, but Heaviside showed retardation effects could change the shape of that field to a stationary observer.

The principle of relativity states that if one is moving with the charge there are no observations that can indicate this motion. However, once again to a stationary observer of the charge moving in an inertial frame, things can change drastically. In 1888 Oliver Heaviside worked out the equation for the electrostatic field of a point charge moving at constant velocity seen by a stationary observer. In modern notation this equation can be written:[42]

$$\vec{\boldsymbol{E}} = \frac{q\left(1-v^2/c^2\right)}{4\pi\varepsilon_o r^3\left[1-\left(v^2/c^2\right)\sin^2\theta\right]^{\frac{3}{2}}}\vec{\boldsymbol{r}}$$

Where θ is the angle between the viewing radius r and the velocity direction v. Note that this equation is the result of electromagnetic retardation and in spite of the similarity to Special Relativity theory is classical and not based upon Einsteinian notions. For low velocities the electric field is seen to attain the value given by using Coulomb's law for stationary charges.

These effects have been worked out in some detail by Jefimenko,[43] which show the effects Heaviside noticed, where the field lines tend to bunch into the equatorial plane perpendicular to the velocity vector and tend to decrease in intensity along the line of motion. But, these effects require relative velocities reasonably near the speed of light. However because of this E field "bunching" any charged particle such as an electron passing through a ring of charge at nearly the speed of light produces an explosive effect on that ring of charge due to the electron E field becoming concentrated like an impulse!

Our current interests, on the other hand, are in elucidating energy flows about a moving charge initially discussed above, which does not require relative motions near light speeds. Given a charge point source, or better a line charge, moving at a constant velocity, we note that this flow of charge represents a current to a stationary observer. And we observe from our causal field equations that this constant current apparently creates a Biot-Savart magnetic field. Since the Electrostatic E field is radial about the

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[42] Rosser , W. G. V., "Classical Electromagnetism via Relativity", New York, 1968, pp. 38-41.
[43] Jefimenko, Oleg D., "Causality, Electromagnetic Induction and Gravitation", Op cit. Appendix 5, pp 175-180.

point charge and the magnetic field in circles about the line of motion, on average, the flow of energy is seen to be in the direction of its motion. Hence as the charge moves forward as we observed in Figure 2, the Poynting vector shows a forward flow of energy in space as well.

Electromagnetic Mass

While this paper deals with Maxwell theory, which regards charge as a continuous fluid described by differential equations, (in the same way that Newton's theory regards mass as a continuous substance) in truth, charge has been found to exist in discrete amounts with each electron having a small fixed amount. And the common electron charged particles also appear to have mass. The ratio of electron charge to mass was determined in a classic experiment by J.J. Thomson in 1897, but then in 1909 Robert Millikan invented an experiment that separated the ratio producing separate numbers for mass and charge of an electron. Millikan employed tiny charged oil drop suspended in an electric field. Thus, gravity acted on the mass causing the drops to fall and the electric field acting on the charge provided a force to hold them up.

However, the above energy flows give us some very interesting speculation. The first being that a charge set in constant velocity motion requires additional energy beyond the kinetic energy due to it's mass that gets stored in a magnetic field. Another interesting feature is that the Poynting vector comprised of the magnetic field and the electrokinetic field as discussed above, shows an energy flow into and out of space. This additional energy is put into space as the charge is accelerated up to its constant velocity and is given up from space when the charge is decelerated back to stationary. This is an interesting effect since there is no evidence that mass is required for charge. To the experimenter, this field energy acts like inertia or kinetic energy. So there is an implication here that even if an electron, for example, had no mass at all, but only charge, it might appear to be a particle possessing mass simply from the electromagnetic effects of relative motion, while Poynting energy flow in the direction of motion represents the kinetic energy is carried along by the particle. However, we are not going to discuss electromagnetic mass. For one thing it cannot be defined for "point charges" as it becomes undefined the way self-inductance does for zero diameter wires. But we will be examining momentum and mass-like effects such as radiation pressure.

Even more interesting in our previous case is the electrokinetic-magnetokinetic Poynting vectors where both increasing and decreasing currents send energy flow into space as radiation. In this situation, it is clear that both acceleration of the charge or deceleration of it results in radiation. Only straight-line constant velocity inertial motion produces no radiation. These results conform to the well-known property of accelerated charges radiating. But a steady current even in a loop does not seem to produce a derivative that is necessary to radiate. And indeed it is observed that DC current loops do not seem to radiate. However, the drift velocity of charges in wire loops is nearly walking speed and hence any radiation due to the acceleration of circular motion would be of too low a frequency to be observed.

Cyclotron Radiation

But it is also well known that charged particles traveling in a circle do indeed radiate producing what is termed "cyclotron radiation". So how can that be? [we are only considering classical cases here rather than synchrotron radiation that occurs from charged particles traveling near the speed of light] It becomes clearer if we have a single charged particle traveling in a circular orbit. Viewed from above the orbit there seems to be no changes in the velocity of the particle except for direction or it's distance to the viewer. Hence we surmise that there is no radiation in that direction. On the other hand, a particle in circular orbits whose position is viewed from the edge of the orbit appears to be undergoing simple harmonic motion back and forth. This is true for all angles from the axis of the circular motion. Thus, such motion would produce an apparent modulated current due to the apparent moving charge and the frequency of the radiation would be given by number of orbits per second as is observed in cyclotron radiation. But it is easily seen that with two particles on opposite sides of the orbit, the radiation from the front side of the orbit is out of phase with that from the backside of the orbit. Hence when one has a great many charged particles, the radiation from one part of the orbit is canceled by the radiation from other particles that is exactly out of phase. The whole effect is most easily understood when the diameter of the charged particles orbit is small enough that retardation over the distance across the diameter of the orbit is negligible. Hence at a great distance which is to say in the "far field" opposite sides of the orbit radiation arrive out of phase and hence cancel showing no transmitted energy, but right at the orbit these fields no not cancel. Since electrokinetic **E** fields produced by acceleration of particles by Lentz' law tend to decelerate accelerating particles this produces the well-known result that much of the energy needed to accelerate particles in a cyclotron is due to the production of cyclotron radiation.

Indeed this radiation near the orbit is discussed in Wikipedia:

Cyclotron radiation has a spectrum with its main spike at the same fundamental frequency as the particle's orbit, and harmonics at higher integral factors. Harmonics are the result of imperfections in the actual emission environment, which also create a broadening of the spectral lines.

Radiation reaction acts as a resistance to motion in a cyclotron; and the work necessary to overcome it is the main energetic cost of accelerating a particle in a cyclotron.[44]

In the physics of electromagnetism, the Abraham–Lorentz force is the recoil force on an accelerating charged particle caused by the particle emitting electromagnetic radiation. It is also called the radiation reaction force.

The formula is in the domain of classical physics, not quantum physics, and therefore, may not be valid at distances of roughly the Compton wavelength or below.[45]

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44 http://en.wikipedia.org/wiki/Cyclotron_radiation

45 http://en.wikipedia.org/wiki/Radiation_reaction

Thus, if one models atoms as electrons in constant velocity orbits about protons, atoms should be expected to radiate and at minimum lose energy and have orbital decay as a result. And of course, they don't, which implies that the something quite different from cyclotron radiation is going on in say Hydrogen with a single presumably circulating electron.

Other Observations

If we examine the various terms for the fields found in the causal equations we notice that electrostatic fields due to charges act essentially radially from the charge to the observer on a line connecting them. Electrokinetic fields are always parallel or anti-parallel to the source current elements and uniformly spherically distributed around the current source elements. And finally all magnetic fields both Biot-Savart and magnetokinetic are the result of a cross product of a unit vector in the direction of the line linking the observer with the direction of the source current elements and source current **J** vector.

The consequence of that last placement is that since the cross product of two parallel vectors is zero there is no magnetic field along the axis of the current. Since we have already noted that electromagnetic power flow takes *both* a magnetic and electric field and since radiation is due to the combination of the magnetokinetic and electrokinetic fields, we immediately conclude that the far field pattern of linear current antenna element is a doughnut shape with no radiation in the direction of the current. And furthermore, we can conclude that the near field inductive magnetic energy of a linear current antenna is also stored in the space surrounding the wire with little found off the ends of the antenna element.

It should be remarked that given the fact that a current-induced magnetic field is zero along the axis of a thin straight current-carrying wire, if one were attempting to apply Faraday's law to explain self-inductance through a change of magnetic flux and some effect of magnetic fields, it would appear that a long straight wire can have no self-inductance. This, of course, has long been experimentally observed to not be the case. The key, being that inductance appears as a result the electrokinetic field created by the current in the wire opposing the current change in the wire and also playing the role of providing energy transport to and from space where inductive energy is stored in the magnetic field about the wire. That the electrokinetic field is created by a changing current, but that $\mathbf{E_K}$ field then cancels part of the electric field driving that current constitutes a feedback situation that has made self-inductance difficult to calculate on a fundamental level.

Kinetic Linear Momentum

In classical mechanics, momentum is defined as mass times velocity of an object or p=mv. Momentum is a vector quantity having magnitude and direction and it is conserved. This means that momentum can change form, but the total quantity of momentum in an isolated closed system always stays the same. Our interest here is in the existence of electromagnetic momentum and it's change of form to mechanical momentum (mass movement) and back.

Newton's Second Law can be written in terms of momentum:

$$F = \frac{dp}{dt}$$

or

$$Change\ in\ Momentum = \Delta p = \int_{t_1}^{t_2} F(t)\, dt$$

Which is the key to electromagnetic momentum since we've already noted that E and B are *force fields.* In a purely mechanical case, *c*onsider what happens if we have a compressed spring in a tube with say a cannonball at the end of the spring. The total momentum of the system is zero initially and there is energy stored in the spring equal to ½ kx^2. Since there is conservation of energy and conservation of momentum, if we trip the spring it will apply a force to the ball ejecting it say to the right with velocity v_1, while Newton's Third law indicates that an equal and opposite force is applies to the tube and spring which recoils to the left with velocity v_2 The ratio of the velocities is the inverse of the ratio of the mass of the cannonball to the mass of the spring and tube. By conservation of energy, the total energy (½ kx^2) is also divided between the two parts;

$$m_1 v_1 + m_2 v_2 = 0 \quad and \quad \tfrac{1}{2} kx^2 = \tfrac{1}{2} m_1 v_1^2 + \tfrac{1}{2} m_2 v_2^2$$

Forces on Charge and Current Distributions

We are aware that the static force on a collection of charges and currents is given by:

$$\vec{F} = \int \left(\rho \vec{E} + \vec{J} \times \vec{B}\right) dv$$

Where ρ is a charge density distribution in space and J is a current density distribution in space and must therefore remain true for time-dependent cases. In other words the instantaneous value of the force on a distribution of charge and current densities is given by the instantaneous value of the E and B fields integrated over those distributions. But to recall the earlier discussion of superposition of fields, it should be noted that E and B fields in the above equation are *partial* fields they represent fields generated by distant charge and current sources. They do not include the fields generated by the charge density distribution ρ and current density distribution J. Furthermore, these distributions are following the Maxwell model of charge and current densities being continuous fluids. Because the field integration is over the charge distributions, point charges or "electrons" will not work because of the "self-inductance" problem of fields becoming "infinite" at small distances.

However, using Maxwell's equations Jefimenko has derived another expression,[46] which we will not derive here, for the time-dependent Maxwell stress equation giving the total force on a collection of charges and currents produced by **E** and **B** fields:

$$\vec{F} = -\varepsilon_o \mu_o \frac{\partial}{\partial t}\int \vec{S}\, dv - \left[\frac{\varepsilon_o}{2}\oint E^2\, d\vec{S} - \varepsilon_o \oint \vec{E}\left(\vec{E}\cdot d\vec{S}\right)\right] - \left[\frac{\mu_o}{2}\oint H^2\, d\vec{S} - \mu_o \oint \vec{H}\left(\vec{H}\cdot d\vec{S}\right)\right]$$

However, in this case the fields in the equation represent the *total* fields which is to say the fields from the distance source as well as those from the charge and current distributions. In these cases the distributions are considered "frozen" and do not react to their own fields. For example a charge distribution does not spread and disperse due to it's own fields. Because these are total fields they represent the total "stress", energy, momentum, etc being put "into" a region of space by the fields.

What we observe here is that the first bracket contains the Maxwell stress equation for the force on a charge distribution for a static **E** field and the second bracket contains the Maxwell stress equation force for a static **H** field on a current distribution. But now there is an additional force term containing the Poynting vector, **S,** and due to the derivative, this new term goes to zero when the fields do not vary in time, yielding the well-known cases for static **E** and **H** fields.

The most amazing thing discovered here is that neither the charge nor the current appears in the force equation! In fact, the force upon a charge distribution in the case of a static E field depends *only* on the value of the **E** field integrated over a closed surface enclosing the charge distribution according to the Faraday-Maxwell stressed medium theory. A similar situation exists for the magnetic stress equation forces. It is remarkable that the force on any charge or current distribution is only dependent upon the conditions in the space around it (the electric and magnetic fields) and not on the distribution itself! And it is even more remarkable that in the static case, the force is dependent only upon field values on an arbitrary closed surface enclosing the charge and current distribution. Fields within the volume of the surface do not even matter in the static case! However, because the fields in question are *total* fields one can see that seeming independence of the charge and current distribution is not quite true. This is because the configuration of the charges and currents experiencing the force is *also* creating fields that are added into the external fields to produce the total field value. Hence changes in the charge and current configuration appear as changes in the value of the integrated fields on the arbitrary surface so that the experienced force does indeed depend on the charge and current distribution even though it is not explicitly used in calculations.

The new time-dependent Poynting term, however, does depend upon the fields within the volume. Note that when there are no charges or currents within the arbitrary surface, the two stress terms in the brackets are zero since they equal the force on charges and currents in our original field force equation, but this is not the case for the remaining

[46] Jefimenko, Oleg, D. , "Electricity and Magnetism", Appleton-Century-Crofts, New York, 1966, P 512.

Poynting term. An interpretation of the Poynting term is that each region of space is subjected to a force equal to minus that term whether or not there are charges or currents in it. Thus, this term represents a force on space itself. Thus, the whole equation can be thought of as the total force on charges and currents is given by the static electric and magnetic values given by the stress equation forces minus the force that is impressed upon space itself.

What this equation and interpretation imply is that if we have charges and currents creating fields in space, those fields act to produce forces on other charge and current distributions. The forces upon the distributions together with their mass represents kinetic energy added to the distributions. However, not all the force created by the fields is going into mechanical motion! A portion is impressed upon space itself as a stress. Thus, when kinetic energy and momentum is imparted to the mass of charge and current distributions by time-variable electric and magnetic fields there is also storage of energy and momentum in space itself, but since this force on space itself is determined by the Poynting vector it is seen that both an electric and magnetic fields are simultaneously necessary for fields to represent stored energy and momentum in space.

It has been shown that either an electric or magnetic field can represent stored energy in space, but a Poynting vector is needed to place or remove that energy from space. Electromagnetic linear momentum on the other hand is proportional to the Poynting vector and hence requires a non-zero Poynting value to even exist.

Storage of Linear Momentum in Space

Since force on an object equals the time rate of change of the momentum on that object, we conclude from our Poynting force term above that any volume of space containing both E and B fields also contains a certain amount of linear momentum given by:

$$\vec{p} = \varepsilon_o \mu_o \int \vec{S}\, dv = \frac{1}{c^2} \int \vec{S}\, dv$$

Where we use p for total momentum and S for the Poynting vector to reduce confusion. Therefore the momentum density (termed g here) or momentum per unit volume (remembering that $c^2 = 1/\mu_o \varepsilon_o$ and $B = \mu_o H$) of fields in vacuum is given by:

$$\vec{g} = \textit{Momentum per unit volume of fields} = \varepsilon_o \mu_o\, \vec{S} = \varepsilon_o \mu_o\, \vec{E} \times \vec{H} = \varepsilon_o\, \vec{E} \times \vec{B} = \frac{1}{\mu_o c^2}\, \vec{E} \times \vec{B}$$

And finally we find the linear momentum density in space where fields are present is proportional to the Poynting Vector, S, as:

$$\vec{g} = \frac{\vec{S}}{c^2}$$

And again we point out that for fields to represent transport of linear momentum *both* electric and magnetic fields must be present so that the Poynting vector is not zero.

Above in Figure 5., we saw the flow of stored energy in space about a charge in linear motion with constant velocity. Now it is seen that those Poynting vectors also represent linear momentum density in space. If we resolve those momentum density vectors into momentum in the direction of motion and perpendicular to it, we find a momentum going out away from the charge behind it and then returning back into the charge in front of it as well as traveling along with the charge. If the charge is stopped or interestingly if one moves in the frame the charge is moving in, then there is no magnetic field and hence no momentum stored in space as $S = 0$. The situation is analogous to observing the momentum of the water in a flowing river, and then being on a raft moving with the river and noting that the water then apparently has no momentum that can seemingly apply forces to your raft even though the momentum of the river with respect to the river bank can propel your boat down river if you stick a paddle into the moving water.

The curved momentum trajectory around the moving charge seems to imply the existence of angular momentum, but careful examination show that by symmetry the momentum flow on the other side of the charge exactly cancels that apparent angular momentum giving a zero total resultant.

A Radiation Pressure Example[47]

It is known that light (or other electromagnetic radiation) produces force on object. It is therefore obvious that the light is delivering momentum at some rate which is equal to the force produced. Light consists of both E and H fields at right angles to each other which clearly represents a Poynting vector and hence momentum density. This example may help clarify these ideas.

In order to make this example a bit more practical and realistic we propose that the situation of interest involves a Mylar balloon in space upon which is shown a large laser beam producing a power of 1000 watts per square meter. The problem is to find the radiation pressure the laser produces on the balloon for both the case where the balloon has a black absorbing surface and also if the balloon were to have a silvery reflecting surface.

First it is important to note that the pressure on the balloon does not depend on the diameter of the balloon. It is only necessary that the laser beam have a diameter large enough that the entire object is covered in radiation. But since the force on the balloon is from pressure or force per unit area, the total force does depend on the area of the balloon intercepting the laser beam.

Having said that, the first task, therefore, would be to calculate the energy density (joules per cubic meter) in the laser beam. We know that each square meter of the cross section of the beam delivers a power of 1000 watts and the speed of light in space is c, so

47 See example 15-5.1, Jefimenko, "Electricity and Magnetism", p. 513.

therefore we first note that since 1 Watt-second equals 1 Joule, the laser beam is delivering 1000 Joules of energy every second. Since that energy is contained in a tube back up the beam the distance that light travels in 1 second, we have 3 x 10^8 meters times 1 meter squared or a volume of 3 x 10^8 cubic meters of fields. Thus the energy density, U, in the laser beam is 333.33 x 10^{-8} Joules per cubic meter or about 3 1/3 microJoules per cubic meter.

Now the momentum density of the radiation is given by:

$$g = momentum\ \ per\ \ cubic\ \ meter\ = \varepsilon_o\ \mu_o\ S$$

And the Poynting vector is given by field energy density times the speed of light or:

$$S = c\,U = \frac{1}{\sqrt{\varepsilon_o \mu_o}} U$$

Since $c^2 = 1/\varepsilon_o\mu_o$ and thus the momentum density per cubic meter in the radiation is:

$$g = \sqrt{\varepsilon_o \mu_o} U$$

Note that here we are assuming the radiation is absorbed. Thus the momentum change goes from the value in the radiation down to zero. If the balloon were a perfect reflector, the momentum would arrive and be reflected in the opposite direction giving double the value for momentum change. The pressure (rate of momentum transfer per unit area) is thus:

$$p = c\ g = \frac{\sqrt{\varepsilon_o \mu_o}}{\sqrt{\varepsilon_o \mu_o}} U$$

Which yields the simple result:

$$p = U = 3\ 1/3\ \ microJoules\ /meter^3$$

Another simple result seen to fall out of the above is the relation:

$$p = \frac{\langle S \rangle}{c}$$

Where <S> is the time averaged pointing vector or in other words the time averaged intensity of the radiation, which is to say our original 1000 watts.

But the units do not seem to relate to a pressure. However we find that the dimensions of these quantities in mass, length and time are as follows:

Energy; Joules; $\mathbf{M\ L^2\ T^{-2}}$

Pressure; Newtons/meter ^{2}; $\mathbf{M\ L^{-1}\ T^{-2}}$

But

Energy /meter^3; Joules/meter^3; $\mathbf{M\,L^2\,T^{-2}\,/\,L^3 \;=\; M\,L^{-1}\,T^{-2}}$

Hence we have calculated a pressure of 3 1/3 microNewtons per square meter, which is quite small. To reiterate, the material question is the difference between a black absorbing material and a silvery shiny one. The answer is that in the former case the momentum comes in and absorbed by the material so our above calculation is correct. But in the case of mirrored material the momentum come in and then is reflected back in the direction from which it came. Since that gives the momentum a negative sign the total momentum exchange is doubled over the previous value or 6 2/3 microNewtons per square meter. Nevertheless, radiation pressure is very real. Mylar “solar sails” or even the balloon experiment has been done in space and the radiation pressure from a laser beam has suspended tiny glass beads.

An Electromagnetic Linear Momentum Example

Consider a charged ring of mass m, symmetrically placed about a length of current-carrying conductor of rotational symmetry as shown in Figure 5. An essential feature of this arrangement is that the magnetic vector potential outside the conductor, and hence the electrokinetic field, is identical to that for a very thin wire. No matter what the diameter of the conductor.[48] However, this does not mean the experiment is electrically independent of the conductor geometry. The inductance of the current-carrying conductor very much depends upon it's geometry and thus so does the amount of magnetic energy stored in space. However for this experiment we shall consider that the conductor is driven by an active current source, which forces a set current in the conductor regardless of it's inductance. This not only forces a set magnetic field in space due to Biot-Savart, but since he electrokinetic field depends only upon he time rate of change of the driving current, if we drive the current say from zero to some final value I, the force on the charged ring will be independent of the conductor geometry since it depends only on the electrokinetic field and the total charge. The ring will fly off the conductor in a direction opposite to the current for rising currents. Since the ring has mass, it's final velocity represents a certain value of momentum acquired by the charged ring.

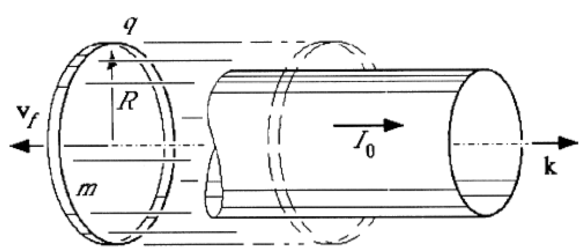

Fig. 3.6 When a current is established in the cylinder, the charged ring flies off the cylinder.

Figure 8. Linear Momentum Experiment (from Jefimenko)

48 See Jefimenko, Oleg D., “Electricity and Magnetism”, example 11-1.2 p.367.

To actually begin to calculate forces, one examines the causal equation giving the electrokinetic field from the flowing current. It is clear that one *could* simply integrate the field caused by the flowing current time rate of change integrating over the total current and the charge on the ring where each charge element dq is experiencing a force from the electrokinetic field of the current density in the conductor. But it is clear that while this is mathematically possible the rotational symmetry of the apparatus suggests there may be a much simpler way to proceed.

Linear Momentum Imparted to the Charged Ring

One can compare Jefimenko's causal equation for an electrokinetic **E** field with the expression for a retarded magnetic vector potential, [**A**], where brackets here denote retarded functions, noting that:

$$\left[\vec{A}\right] = \frac{\mu_o}{4\pi}\int\frac{\left[\vec{J}\right]}{r}dv = \frac{1}{4\pi\varepsilon_o c^2}\int\frac{\left[\vec{J}\right]}{r}dv$$

Where [**J**] is the retarded current density and the integral is over the source current. On the other hand, from the causal Maxwell equations for the Electrokinetic **E** field we have the very similar relationship:

$$\vec{\boldsymbol{E}}_{\boldsymbol{K}} = -\frac{1}{4\pi\varepsilon_o c^2}\int\frac{1}{r}\left[\frac{\partial\vec{\boldsymbol{J}}}{\partial t}\right]dv'$$

We note that if the structure is physically small enough, the circuit approximation can be used which is to assume the current changes simultaneously at any position of the current and if the distances to the induced emf are short enough that transmission times are negligible, retardation can be ignored. In fact, Jefimenko has shown that when $\mathbf{E_K}$ is linearly rising or falling in time, replacing the retarded magnetic vector potential with the ordinary (unretarded) **A** results in an exact solution with no approximations[49]. Thus, one can bring the time derivative outside the integral and quickly arrive at a simple law of induction:

$$\vec{\boldsymbol{E}}_{\boldsymbol{K}}(xyz) = -\frac{\partial\vec{\boldsymbol{A}}(xyz)}{\partial t} \quad where\ \ emf = \oint E_K\cdot dl$$

where the emf integral is around a secondary induced emf loop in a typical case of induction and retardation is neglected for the derivative yielding the simple relationship:

$$\vec{\boldsymbol{E}}_{\boldsymbol{K}} = -\frac{\partial\vec{\boldsymbol{A}}}{\partial t}$$

[49] Jefimenko, Oleg D., "Causality Electromagnetic Induction and Gravitation" ` 2nd Edition. Electret Scientific Co. Star City, WV, 2000, sec 2-4, pp. 31-33.

Note that the $\mathbf{E_k}$ field vector is parallel or anti-parallel to **A** at that location and that **A** is determined by the sum of all vectors representing current elements divided by the distance to them. In other words every **A** contribution is parallel to the vector direction of it's current source. Hence the electrokinetic $\mathbf{E}_k$ field will tend to be opposite the direction of the current for rising currents and in the direction of the current for falling currents. For, example in the case of a straight wire, each current element dI in that wire creates an electrokinetic field about that element that is a uniform spherical distribution of field vectors all pointing in the direction of the current. The total electrokinetic field is the sum of all those contributions.

Another observation is that the above relationship between the electrokinetic field and the time rate of change of the vector magnetic potential does not involve the **B** field except by it's relationship to the curl of **A**. Thus, if **A** has no curl the electrokinetic $\mathbf{E_K}$ field can appear to exist where there is no **B** field which is to say where the Poynting vector is zero. This is well known outside long solenoids and toroids. So in the same way that we found that force on a static charge and current distribution can be found either by an integration of fields over the volume of the distribution *or* by a surface integral of all fields over a volume enclosing the distribution, here we shall find that the electrokinetic field force and it's momentum is "measured by" the vector magnetic potential where the magnetic field **B** is "measured by" the **A** field. Note that $\mathbf{E_K}$, **A**, and **B** are all retarded and do not "cause each other" in any sense.

Therefore by Newton's Second Law written in terms of momentum:

$$\vec{F} = \frac{d\vec{p}}{dt} = q\ \vec{E}_K$$

or in general when $\mathbf{E_K}$ varies over the charge distribution:

$$\Delta P = \int F\ dt = \iint \rho\ E_K\ dv'\ dt$$

But in our case here, because of symmetry, Both **A** and thus $\mathbf{E_K}$ are a constant value over the ring and thus since they depend only on t, can be factored out of the double integration leaving the integration of the charge density over the volume which simply yields charge. Expressed in terms of the magnetic vector potential, we then have

$$Change\ in\ Momentum = \Delta\vec{\mathbf{p}} = \int_{t_1}^{t_2} F(t)\ dt = -q\int_{t_1}^{t_2} \frac{\partial\vec{\mathbf{A}}}{\partial t}\ dt$$

Thus the change in mechanical momentum imparted to the ring is given by where we note we started from zero current so $\mathbf{A_1 = 0}$.

$$\Delta\vec{\mathbf{p}} = \int_{t_1}^{t_2} F(t)\ dt = -q\int_{t_1}^{t_2} \frac{\partial\vec{\mathbf{A}}}{\partial t}\ dt = -q\ (\vec{\mathbf{A}}_2 - \vec{\mathbf{A}}_1) = -q\ \Delta\mathbf{A} = -q\,\mathbf{A}_{Final}$$

Thus to compute the kinetic momentum imparted to the ring we need only calculate the magnetic vector potential at the ring (which is uniform over it) for our final current I and multiply by -q.

The magnetic vector potential for a thin wire of length 2L at radius r is well known and given by:

$$\vec{A}=\frac{\mu_o I}{2\pi}\ln\frac{\left(L+\sqrt{L^2+r^2}\right)}{(r)}\vec{k}$$

Where **k** is a unit vector in the direction of the current, which if the wire is long compared to the radius of the ring (L>>r) simplifies to:

$$\vec{A}=\frac{\mu_o I}{2\pi}\ln\frac{2\mathrm{L}}{(r)}\vec{k}$$

And thus the linear momentum imparted to the ring is given by

$$\vec{p}=-q\frac{\mu_o I}{2\pi}\ln\frac{2\mathrm{L}}{(r)}\vec{k}$$

Which gives a final velocity (p = mv) of:

$$\vec{v}=-\frac{q\mu_o I}{2\pi m}\ln\frac{2\mathrm{L}}{(r)}\vec{k}$$

Where it is seen the velocity of the ring is in the opposite direction to the rising current in the conductor. But observe that the final result is not quite correct. The velocity of the charged ring rises from zero to some final value, which represents an increasing current in the opposite direction to the driving current. This changing current therefore produces an Electrokinetic $\mathbf{E_K}$ field by virtue of it's motion. This new field acts not only on the charged ring itself reducing the driving force, but also on the current in the original tube.

Since the apparatus began with zero momentum, conservation of momentum indicates that momentum must somehow always be zero. The charge on the ring is radial and cannot impart any forces in the direction ring motion. One might suspect that since the moving charged ring constitutes an electric current perhaps that is the source of a reaction force back upon the driving conductor. But the ring motion is parallel to the wire and although it does constitute an additional current, that current is parallel to the driving current. As is well known parallel currents can attract or repel each other, but again that direction is perpendicular to the momentum imparted to the ring and also is symmetrical about source conductor and hence is of no effect. Therefore our sole possibility is that the missing oppositely directed momentum was stored in space. This concept of electromagnetic momentum stored in space is the point of the Feynman Paradox example we will be discussing below.

Linear Electromagnetic Momentum Stored in Space

As it turns out we already know how to calculate the amount of electromagnetic momentum stored in all of space. We simply integrate the Poynting vector created by the fields of our apparatus over all space.

It was previously shown that the amount of linear momentum stored in a given volume of space is given by:

$$\vec{\boldsymbol{p}} = \varepsilon_o \mu_o \int \vec{\boldsymbol{E}} \times \vec{\boldsymbol{H}} \; dv$$

And the task is to relate this momentum to the magnetic vector potential, A. Jefimenko's full derivation will be shown here being as typical of his work. [50]

Since

$$\vec{\boldsymbol{B}} = \mu_o \vec{\boldsymbol{H}} \;\; and \;\; \vec{\boldsymbol{D}} = \varepsilon_o \vec{\boldsymbol{E}} \;\; and \;\; \vec{\boldsymbol{B}} = \nabla \times \vec{\boldsymbol{A}}$$

We obtain

$$\vec{\boldsymbol{p}} = \int \vec{\boldsymbol{D}} \times \left(\nabla \times \vec{\boldsymbol{A}}\right) dV$$

The following vector identity is then applied to that equation:

$$\oint \left(\vec{\boldsymbol{X}} \cdot \vec{\boldsymbol{Y}}\right) d\,\vec{\boldsymbol{S}} - \oint \vec{\boldsymbol{Y}} \left(\vec{\boldsymbol{X}} \cdot dS\right) - \oint \vec{\boldsymbol{X}} \left(\vec{\boldsymbol{Y}} \cdot d\,\vec{\boldsymbol{S}}\right) =$$
$$\int \left[\vec{\boldsymbol{X}} \times \left(\nabla \times \vec{\boldsymbol{Y}}\right) + \vec{\boldsymbol{Y}} \times \left(\nabla \times \vec{\boldsymbol{X}}\right) - \vec{\boldsymbol{X}} \left(\nabla \cdot \vec{\boldsymbol{Y}}\right) - \vec{\boldsymbol{Y}} \left(\nabla \cdot \vec{\boldsymbol{X}}\right)\right] dV$$

Where "X" = D and "Y" = A.

$$\oint \left(\vec{\boldsymbol{D}} \cdot \vec{\boldsymbol{A}}\right) d\,\vec{\boldsymbol{S}} - \oint \vec{\boldsymbol{A}} \left(\vec{\boldsymbol{D}} \cdot d\,\vec{\boldsymbol{S}}\right) - \oint \vec{\boldsymbol{D}} \left(\vec{\boldsymbol{A}} \cdot d\,\vec{\boldsymbol{S}}\right) =$$
$$\int \left[\vec{\boldsymbol{D}} \times \left(\nabla \times \vec{\boldsymbol{A}}\right) + \vec{\boldsymbol{A}} \times \left(\nabla \times \vec{\boldsymbol{D}}\right) - \vec{\boldsymbol{D}} \left(\nabla \cdot \vec{\boldsymbol{A}}\right) - \vec{\boldsymbol{A}} \left(\nabla \cdot \vec{\boldsymbol{D}}\right)\right] dV$$

We observe that since all fields tend to zero at great distances and since our integration is over all space, the surface integrals in the top line are all zero. Thus:

$$\vec{\boldsymbol{p}} = \int \vec{\boldsymbol{D}} \times \left(\nabla \times \vec{\boldsymbol{A}}\right) = \int -\vec{\boldsymbol{A}} \times \left(\nabla \times \vec{\boldsymbol{D}}\right) + \vec{\boldsymbol{D}} \left(\nabla \cdot \vec{\boldsymbol{A}}\right) + \vec{\boldsymbol{A}} \left(\nabla \cdot \vec{\boldsymbol{D}}\right) dV$$

Since by Maxwell's equations we can write:

$$\nabla \cdot \vec{\boldsymbol{D}} = \rho \;\; and \;\; \nabla \times \vec{\boldsymbol{E}} = -\frac{d\,\vec{\boldsymbol{B}}}{dt} \; and \;\; \vec{\boldsymbol{D}} = \varepsilon_o \vec{\boldsymbol{E}}$$

And since once momentum is stored in space E and H are time independent, □ x D = 0 we now have:

[50] Jefimenko, Oleg D., , "Electromagnetic Retardation and the Theory of Relativity", Electret Scientific, 2004, pp. 187-188.

$$\vec{\boldsymbol{p}} = \int \vec{\boldsymbol{D}} \times \left(\nabla \times \vec{\boldsymbol{A}}\right) = \int + \vec{\boldsymbol{D}} \left(\nabla \cdot \vec{\boldsymbol{A}}\right) + \vec{\boldsymbol{A}}\ \rho\ dV$$

and since again fields are not changing with time once storage is established:

$$\vec{B} = \mu_o \vec{\mathbf{H}} \ \ and \ \ \nabla \cdot \vec{\mathbf{A}} = -\varepsilon_o\ \mu_o \frac{\partial \varphi}{\partial t}$$

the second half of the identity becomes:

$$\vec{\boldsymbol{p}} = \int \vec{\boldsymbol{D}} \times \left(\nabla \times \vec{\boldsymbol{A}}\right) = \int \vec{\boldsymbol{A}}\ \rho\ dV$$

And since at this point we observe that this integral is only non-zero where there is charge density, or in other words is only over the charge on the ring and since in our experiment the vector magnetic potential is uniform over the ring, A can be factored out and the remaining integral is simply that which gives the total charge “q” on the ring or:

$$\vec{\boldsymbol{p}} = q\ \vec{\boldsymbol{A}}$$

Which is our final result for the electromagnetic momentum stored in all space, which it turns out rather unsurprisingly is the same value as that found imparted mechanically to the ring, except that this time the momentum is in the direction of the source current rather than opposite to it. Hence the total linear momentum remains zero in spite of the ring being accelerated and linear momentum is conserved.

A very interesting variation of this example would be to actually attach the ring by insulators to the driving wire. This would increase the apparent mass accelerated by the electrokinetic field to include that of the driving wire, but there still would be the same linear momentum imparted to the entire apparatus. It would appear that a propulsion has been achieved with no reaction except against space itself which does not move.

Einstein has suggested that such a thing does not occur because of a change of mass due to emission of electromagnetic energy and that the mass-energy of a system is conserved. However, our purpose here is to examine electromagnetic energy and momentum from a strictly classical viewpoint and therefore such relativistic considerations will not be examined.

Linear Electromagnetic Momentum: Another Calculation

The redundancy of electromagnetic calculations has been shown, and it is therefore no surprise that other expressions for stored linear momentum exist. This expression similar to the one just derived relating to charge and the magnetic vector potential, was part of a relativistic discussion by Furry[51], however Kirk T. McDonald has provided a summary derivation in classical electromagnetism that we present below.[52]

As noted above in our discussion of energy stored in space, some materials have conductivity and as our conductor in this example exhibits a potential variation in space as a result of the voltage-drop from currents passing through it. Since energy flows through the material and energy can be stored within it, it is logical that linear momentum can be stored as well.

Therefore to calculate the linear momentum stored in space due to currents and potential in our conductor, we begin with the usual expression that integrates the Poynting vector over all space giving the electromagnetic linear momentum.

$$\vec{\boldsymbol{p}} = \varepsilon_o \mu_o \int \vec{\boldsymbol{S}}\, dv = \varepsilon_o \int \vec{\boldsymbol{E}} \times \vec{\boldsymbol{B}}\, dv = -\varepsilon_o \int \nabla \Phi \times \vec{\boldsymbol{B}}\, dv$$

Where we have used the relationship:

$$\vec{\boldsymbol{E}} = -\nabla \Phi$$

using the vector identity:

$$\nabla \Phi \times \vec{\boldsymbol{X}} = \nabla \times \Phi \vec{\boldsymbol{X}} - \Phi \nabla \times \vec{\boldsymbol{X}}$$

There is obtained:

$$\vec{\boldsymbol{p}} = \varepsilon_o \int \Phi \nabla \times \vec{\boldsymbol{B}}\, dv - \varepsilon_o \int \nabla \times \Phi \vec{\boldsymbol{B}}\, dv$$

since

$$\nabla \times \vec{\boldsymbol{H}} = J \text{ so } \frac{\nabla \times \vec{\boldsymbol{B}}}{\mu_o} = \vec{\boldsymbol{J}}$$

we have:

$$\vec{\boldsymbol{p}} = \varepsilon_o \mu_o \int \Phi \vec{\boldsymbol{J}}\, dv - \varepsilon_o \int \nabla \times \Phi \vec{\boldsymbol{B}}\, dv$$

Using the vector identity:

$$\oint \vec{\boldsymbol{X}} \times d\vec{\boldsymbol{S}} = -\int \nabla \times \vec{\boldsymbol{X}}\, dv$$

where **“X” = Φ B,** thus:

[51] Furry, W. H., “Examples of Momentum Distribution in the Electromagentic Field and in Matter”, Am. J. Phys. **37**, 621, (1969).
[52] McDonald, Kirk T. , “Momenum in a DC circuit”, Joseph Henry Laboratories, Princeton University, Princetom NJ, 08544, (May 26, 2006) see: http://puhep1.princeton.edu/~mcdonald/examples/loop.pdf

$$\vec{p} = \varepsilon_o \mu_o \int \Phi \vec{J}\ dv + \varepsilon_o \oint \Phi \vec{B} \times d\,\vec{S}$$

Thus as usual since the surface integral is over all space and B equals zero at infinity, and with the identity $\varepsilon_o\,\mu_o = 1/c^2$ the result is obtained.

$$\vec{p} = \frac{1}{c^2} \int \Phi \vec{J}\ dv$$

Some comment is required on this result.

Note that the integral is zero where **J** equals zero so that the integration is only over the current-carrying material. In our example the charged cylinder appears nowhere in this result. Hence this linear momentum is *not* related to the linear momentum found above. Therefore this result represents linear momentum stored *within* the conductor, which was ignored in the magnetic vector potential calculation and essentially represents linear electromagnetic momentum created by the conductor on itself! Hence this is an *additional* momentum and not a redundant calculation of the momentum stored outside the conductor.

McDonald in a note[53] makes the comment:

"The third form of eq. (1) indicates that if the magnetic field is created by steady currents in a good/super-conductor, over which the scalar potential Φ is constant, then

$$\vec{p} = \frac{\Phi}{c^2} \int \vec{J}\ dv = 0$$

Hence, we restrict our attention to magnetic fields created by structures that need not be equipotentials, such as permanent magnets made from ferromagnetic grains embedded in a nonconducting matrix."

The point being made here is that since the current density vector **J** of stationary currents is solenoidal, the integral of that field over the volume is zero.[54] This seems to imply that super-conductors or very good conductors cannot create electromagnetic momentum. However, our example as well as the Feynman paradox (which uses a super-conducting loop) clearly shows this is not the case. What is shown is that for very good conductors (as in our source wire) the source current for the electrokinetic driving fields does not impart momentum to the driving conductors *themselves* provided conductivity is high. This makes sense since in excellent conductors positive and negative charges are in balance, hence by our previous magnetic vector potential relationship we see that the

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[53] McDonald, Kirk T., "Electromagnetic Momentum of a Capacitor in a Uniform Magnetic Field", Joseph Henry Laboratories, Princeton University, Princeton, NJ 08544 (June 18, 2006; revised January 5, 2007) see section: 2.1; http://cosmology.princeton.edu/~mcdonald/examples/cap_momentum.pdf.

[54] Plonsey and Colin, "Principles and Applications of Electromagnetic Fields" , Mc Graw Hill 1961, p. 170.

effective charge is zero and hence the imparted and stored momentum is also zero. And another argument would be that when materials become super-conducting all magnetic fields even static ones are driven from the material. Hence, no Poynting vector and hence no stored momentum can exist within the superconducting conductor. The similarity of our example to a coaxial cable suggests an examination of that situation as well.

Momentum in a Coaxial Cable

The linear momentum in a coaxial cable bearing currents has already been examined by McDonald.[55] He examines a length of coaxial cable with an inner conductor of radius, a , and a non-zero resistivity, ρ, and an outer cylindrical conductor of radius b, with zero or very low conductivity. The cable is driven at one end by a battery and terminated at the other with a resistance R_O. The magnetic field for such an arrangement is known and B is zero outside the outer conductor. Also the magnetic vector potential is known and easily determines the electrokinetic field at either conductor.

The difference here from the above example is that the charges are free to move on the conductors and not attached to the material as in our charged ring case. As a result, an electrokinetic field creates or modifies the currents rather than producing forces upon the mass. All this is no surprise since such effects are termed inductance and are well known for coaxial cables.

Thus, we expect no mechanical momentum to appear in the cable when conductors are very good or super-conducting. Therefore with no mechanical momentum appearing we expect no electromagnetic momentum (what McDonald calls "hidden" momentum) to balance it. But since the McDonald example assumes a non-zero resistivity we know from our previous calculation for linear momentum using the electric potential and conductivity, a small value may appear which McDonald calculates and then conserves using a change of mass.

Our point here, is that classically, a coaxial conductor system constructed of very low or superconducting materials will not produce either mechanical momentum nor electromagnetic "hidden" momentum stored in space.

An Example of Conservation of Linear Momentum

At this point it will be instructive to consider an example that demonstrates conservation of linear momentum that includes the "hidden" momentum that is stored in space electromagnetically. This example will be synthesized out of a number of calculations found in various Jefimenko textbooks. This example will consider the fields, forces, energy and momentum involved in flat ribbon conductors as well as a pair of such ribbons formed into a parallel ribbon transmission line circuit.

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[55] McDonald, Kirk. T., " 'Hidden' Momentum in a Coaxial Cable", Joseph Henry Laboratories, Princeton University, Princeton, NJ 08544 (March 28, 20062); http://cosmology.princeton.edu/~mcdonald/examples/hidden.pdf

We begin by calculating the magnetic fields produced by a flat ribbon “bus bar” having a current density J and therefore carrying a current I. A simple application of Ampere’s law to this is seen as diagramed in Fig. 6, which shows two calculations for the magnetic field at the center of the ribbon equally spaced from the edges both inside and outside (but close to) the conductor. [56] The conductor is taken to be much longer than it’s width.

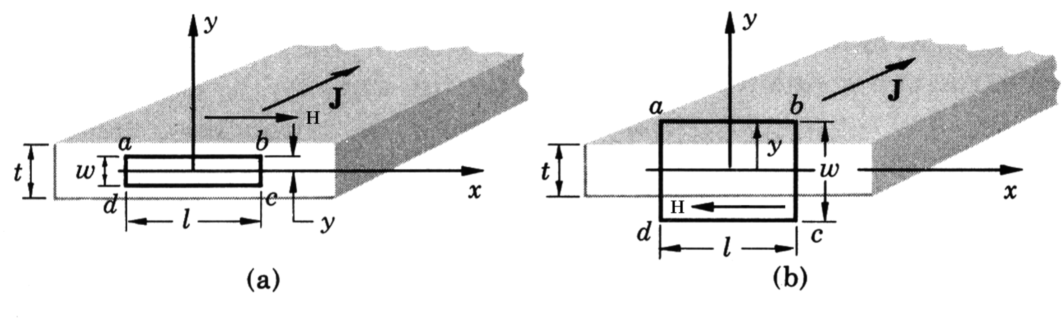

Figure. 9. Calculation of Magnetic Field of Current-Carrying Slab.

Since we are interested in the magnetic field at a position centered between the edges of the bar that are far away and very close to the surface of the bar, the magnetic field will be uniform, tangential to the bar and by the right hand rule perpendicular to the current. The magnetic field will be in the + x direction above the bar centerline and in the –x direction below the centerline. Ampere’s law is written:

$$\int \vec{H} \bullet d\vec{l} = \int \vec{J} \bullet d\vec{S}$$

Inside the bar, the loop and surface in question is determined by the rectangle a,b,c,d, but outside the bar the surface determining the current integral is given by the end of the bar which is to say l times t. Since the magnetic field, **H**, is perpendicular to the sides of the loop, only the top and bottom of the rectangle contribute to the integral of the magnetic field around the loop or:

$$\int \vec{H} \bullet d\vec{l} = \int_a^b \vec{H} \bullet d\vec{l} + \int_c^d \vec{H} \bullet d\vec{l} = 2Hl$$

Inside the bar :

$$I = \int \vec{J} \bullet d\vec{S} = 2ylJ \quad Where \ y \leq \frac{t}{2}$$

Thus:

$$H_x = yJ \ Where \ y \leq \frac{t}{2}$$

Outside the bar the surface integration is limited to the end of the bar and in this case there is found that the **H** field is given by:

[56] Jefimenko, Oleg D, “Electricity and Magnetism", Appleton-Century-Crofts, NY, 1966, p. 333.

$$H_x = \frac{1}{2} Jt \frac{y}{|y|} \;\; Where \;\; y \geq \frac{t}{2}$$

We observe that the sign of H flips at the centerline where y = 0 and that at y == t/2 which is at the surface of the conductor we have a tangential field given by:

$$H_x = \pm \frac{Jt}{2} \;\; Where \;\; y = \frac{t}{2}$$

And where the magnetic field is tangential and in the + x direction on the top of the bar and tangential and in the – x direction on the bottom side.

For completeness we can expand on this solution to find the field about such a thin ribbon rather than just in its center and close to the surface. Once again the ribbon is considered to be much longer than the distance away from it where fields are measured.

This solution as provided by Jefimenko[57] involves a relatively extended derivation starting with a calculation of the magnetic field from a surface current density, $J_{surface}$ integrated by a line integral rather than the J_{volume} where current is calculated by a surface integral as above. We will not give this derivation here, as these calculations will be derived in the next paper in a discussion of Huygens principle and diffraction of electromagnetic waves. Therefore we will only give the final results here, postponing the proof of these relationships to that later time. To understand these results the following diagram provides the various quantities:

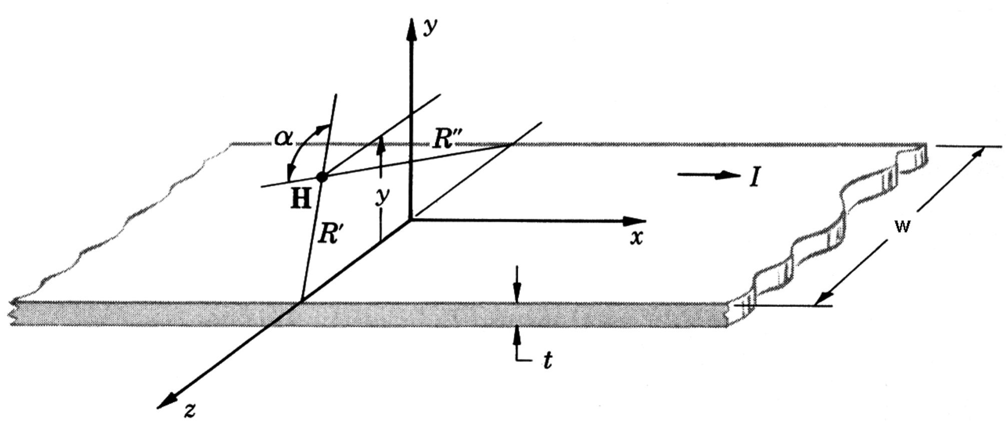

Figure 10. Magnetic field about a bus bar in the x direction measured at a distance y and z.

A simplifying feature of this case is the angle alpha formed by two lines from the observation point to the edges of the bar. Here we calculate both the usual tangential $\mathbf{H_z}$ as well as a perpendicular $\mathbf{H_y}$ that appears toward the edges of the thin ribbon. Again, the key assumptions here are that the ribbon is much longer than the distance to our field measuring position and that the ribbon is very thin which is to say that t is very small.

[57] Jefimenko, Oleg D, "Electricity and Magnetism", Appleton-Century-Crofts, NY, 1966, p. 354-356.

The tangential field $\mathbf{H_z}$ can be shown equal to:

$$H_z = \frac{Jt}{2\pi}\alpha$$

And we observe that in the center of the ribbon close to the surface where y and z equal zero, $\alpha = \pi$ and the same result as previously calculated is obtained, while off the edge of the ribbon, $\alpha = 0$ and hence there is no z component to the magnetic field.

In this case with a ribbon of width w we know that the current in that ribbon is related to the current density in it by the formula: J = I / tw thus for a ribbon of width w carrying a current I, we have:

$$\vec{H}_z = \frac{I}{2\pi w}\alpha$$

and again we observe that near the center of the ribbon where $\alpha = \pi$ the field is tangential and does not depend on the distance from the surface for small distances where $\alpha \approx \pi$.

For the perpendicular magnetic field $\mathbf{H_y}$ it can be shown to be given by:

$$\vec{H}_y = \frac{Jt}{2\pi}\ln\frac{R'}{R''}$$

or

$$\vec{H}_y = \frac{I}{2\pi w}\ln\frac{R'}{R''}$$

note that at the centerline of the ribbon R' / R'' = 1 and ln (1) = 0 which verifies our original assumption that the magnetic field in the center of the ribbon even at a distance is purely tangential as $\mathbf{H_y} = 0$.

At this point we can begin to build our experimental apparatus that will consist of two current carrying ribbons arranged as in Fig. 8.

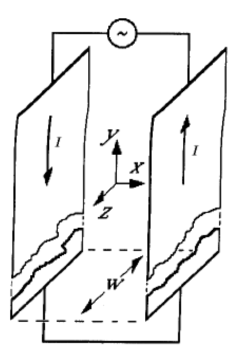

Figure 11. Two closely spaced current carrying ribbons.

The first task is to find the magnetic field in the region centered between the ribbons. Since as we have observed the field direction is reversed between the top and bottom of a current-carrying ribbon and because the current direction is reversed between the two ribbons, the magnetic field between the ribbons add and the total field is given by:

$$\vec{H}_z = \frac{I}{w}\,\vec{u}_z \quad or \quad \vec{B}_z = \frac{\mu_o I}{w}\,\vec{u}_z$$

Where $\mathbf{u_z}$ is a unit vector in the z direction.

However our real goal is to calculate the electrokinetic field produced near the origin located centered between the ribbons when there is a time rate of change in the current. In order to do that we first must find the magnetic vector potential at that location. We are familiar with the equation:

$$\vec{B} = \nabla \times \vec{A}$$

And in Cartesian coordinates the curl of A is given by:

$$\nabla \times \vec{A} = \vec{a}_x\left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right) + \vec{a}_y\left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}\right) + \vec{a}_z\left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right)$$

But we are already aware that the B field only has a component in the z direction, which just leaves us the last term in the above equation. Furthermore, we know that the direction of A is always in the same direction as the current element producing it, so A can have no component in the x direction.

This leaves us with the simple differential equation:

$$B_z = \frac{dA_y}{dx}$$

Integrating both sides and inserting the value of B_z we obtain:

$$A_y = \frac{\mu_o I}{w}x + C$$

We choose the constant of integration to be zero so that $A_y = 0$ at the origin centered between the two ribbons. From this it is seen that the Electrokinetic E field between the ribbons is given by:

$$E_y = -\mu_o \frac{\partial I}{\partial t}\frac{x}{w}$$

Where w is the ribbon width and x is distance from origin centered between ribbons.

Thus, the electrokinetic E field is in opposite directions on opposite sides of the origin. Hence a positive test charge on the side of the upward flowing current will experience an upward force if the current decreases while the same test charge near the other ribbon will experience a downward force in the same conditions. Thus we see that the further charges are from the centerline and the closer to the ribbons the more force they experience as a result of changing currents in the ribbons.

And installing such charges is exactly what we intend to do. We now modify our apparatus by installing a capacitor between the ribbons as shown in Fig. 9.

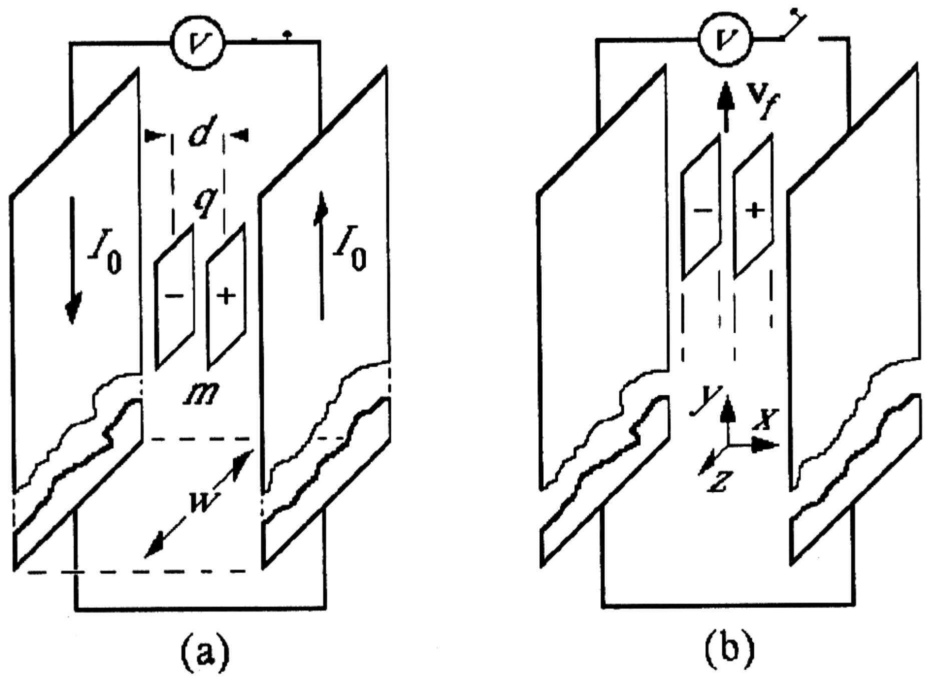

Figure 12. Impulse forces on a charged capacitor between current-carrying ribbons.

This problem posed by Jefimenko[58] involves a capacitor placed so the negative plate is near the downward current, but also where the current begins in a flowing state and then is suddenly stopped. These two facts cause a double reversal from the test charge we just discussed so that force on that plate is upward. The other capacitor plate has the opposite charge (+) reversing the force, but is on the other side of the centerline where the electrokinetic field is in the opposite direction. Therefore BOTH plates of the capacitor experience an upward force when the current is switched off as is shown in (b). Since the thin capacitor plates have a total mass, m, a certain final velocity (and momentum) can be calculated.

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[58] Jefimenko, Oleg D., "Causality Electromagnetic Induction and Gravitation" ` 2nd Edition. Electret Scientific Co. Star City, WV, 2000, pp. 51-52.

Note that like the Feynman Paradox[59] which we will discuss below that involves angular momentum, here we also have a problem given that the capacitor ends up with a final velocity, v, and hence a momentum mv, whereas the apparatus began sitting there not moving at all with zero apparent momentum. There is, therefore, an apparent violation of conservation of linear momentum. But, by now we are well aware that there is "hidden" momentum stored in space by electromagnetic action that we can calculate.

We have seen above [page 55] that the following relation holds when the electrokinetic E field is uniform over our charge distribution (plates are thin) and can be factored out of the double integral when E_k is only a function of time and not position.

$$\Delta\vec{p} = \int_{t_1}^{t_2} F(t)\, dt = -q\int_{t_1}^{t_2} \frac{\partial\vec{A}}{\partial t}\, dt = -q\left(\vec{A}_2 - \vec{A}_1\right) = -q\Delta\vec{A}$$

Thus operating on just the right side plate (the total final momentum of the capacitor is the sum of both plates).

$$\Delta\vec{p}_{one\ plate} = \frac{mv}{2} = -q\Delta A = \frac{q\mu_o I_o d}{2w}\vec{u}_y$$

or the final velocity is given by:

$$\vec{V}_{fnal} = \mu_o \frac{qdI_o}{mw} u_y$$

And we find the moving capacitor attains a final *mechanical* momentum value of

$$P = \mu_o \frac{qdI_o}{w}$$

At this point our task is now to try to find the missing momentum represented by the final velocity of the capacitor and discover where it came from. We showed above [page 50] that any volume of space containing E and H fields also contains an amount of linear momentum given by:

$$\vec{p} = \varepsilon_o\mu_o \int \vec{S}\ dv = \frac{1}{c^2}\int \vec{E}\times\vec{H}\ dv$$

We have already seen that H is uniform between the center parts of the ribbons. Since the capacitor plates have charges, there is an E field between the plates. We assume the ribbons are powered by "floating" power supplies so that the potential of each ribbon is adjusted to match the potential of the nearby moving plate. Currents however match. Thus, the E field is zero outside the moving capacitor between the current-carrying ribbon and the charged plate. Hence the only contribution to the final momentum can come from the E field between the two capacitor plates.

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[59] Feynman, Leighton, Sands, "The Feynman Lectures on Physics", Addison-Wesley Co. Palo Alto, 1963, Section 17-4 Volume II.

Using our previous results for the H field between the ribbons and the standard calculation for a parallel plate capacitor E field. We find:

$$\vec{H} = \frac{I_o}{w}\vec{u}_z$$

and

$$\vec{E} = \frac{q}{\varepsilon_o A}u_x$$

Where A is the area of the plates.

From these we can use the above equation to calculate the electromagnetic momentum stored within the capacitor.

$$\vec{p}_{EM} = \varepsilon_o \mu_o \int \frac{I_o}{w}\frac{q}{\varepsilon_o A}dv = \varepsilon_o \mu_o \frac{I_o}{w}\frac{q}{\varepsilon_o A}\int dv = \mu_o \frac{qI_o}{wA}(dA) = \mu_o \frac{qdI_o}{w}$$

Since the current in the ribbons goes to zero, the magnetic field goes to zero and the final value of stored momentum is thus, zero. Thus, the above value represents the total value of the change in electrical momentum stored in the space between the capacitor plates. It is seen to be identical in value to the mechanical momentum attained by the capacitor due to its velocity gained from the changing current in the ribbons.

Therefore it is easily seen that our apparent violation of conservation of linear momentum has simply missed the fact that linear momentum can be stored in space through the use of electric and magnetic fields and that space can give up that momentum to propel an object to some velocity where that final velocity represents a mechanical momentum equal to the amount of electrical momentum that the space gave up. Note that if the ribbons were grounded, for example, additional capacitors are formed between plates and ribbons that store more momentum and but that momentum is in the opposite direction and balances the "reaction" momentum on the ribbons from the charges that appear there due to the outside capacitors. When we matched potentials, since V =0, q = cV = 0.

This interesting result that may be of some philosophical interest falls straight out of classical Maxwellian electromagnetic theory by relatively simply calculations.

Mass-Energy Relationship

While the purpose of this paper does not include a discussion of electromagnetics in relation to relativity, it is instructive to consider the following calculation. Jefimenko has shown that given a laboratory frame with particle moving in the x direction in a "primed" frame at velocity v, the energy and momentum transform as follows:[60]

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[60] Jefimenko, Oleg D., , "Electromagnetic Retardation and the Theory of Relativity", Electret Scientific, 2004, ff. 198

$$p_x' = \gamma \left[p_x - \left(\frac{v}{c^2} \right) W \right]$$

where W is energy and p momentum and γ is the usual relativistic abbreviation:

$$\gamma = \frac{1}{\left(1 - \frac{v^2}{c^2}\right)^{\frac{1}{2}}}$$

But since our primed frame is moving with the particle that momentum is zero, which gives the equation:

$$W = \frac{c^2 p}{v}$$

Now since the relativistic momentum of a particle moving with velocity u is given by:[61]

$$p = \gamma\ mv$$

so

$$W = \frac{mc^2 v}{v\left(1 - \frac{v^2}{c^2}\right)^{\frac{1}{2}}}$$

and since at rest v = 0 we find that the "rest energy" is given by

$$W = mc^2$$

In 1881 J.J. Thomson concluded that charged bodies had additional mass due to the electrostatic fields from the charge.[62] Today, however it is recognized the most self-energy is not electronic but nuclear self-energy.

Newton's Third Law

Before continuing to a discussion of the transmission and storage of angular momentum, an examination of Newton's Third Law is in order. Newton expressed it thus: "*To every action there is always opposed an equal reaction. Or, the mutual actions of two bodies on each other are always equal, and directed to contrary parts.*" As we have already noted, in electrostatics such a rule seems to always be true. Given two separate charge distributions A and B, we find that A imposes an E field E_A on charges B and B creates an E field E_B imposing a force on A. As expected, these are equal and opposite actions.

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61 see footnote 43

62 Thomson, J.J. "On the electric and magnetic effects produced by the motion of electrified bodies", Philos. Mag. **11**, 229-249, (1881).

But we have already noticed that if one of the charge distributions suddenly moves, because of the distance separating them, there will be a period of time while if A moves, B will not have sensed the motion and forces will be unbalanced. But there is more to this than retardation effects. A moving charge constitutes a current. And as that current changes increasing up from zero an electrokinetic E field is created at the other charge, which is stationary and therefore cannot create a current! Thus, while electrostatic forces are balanced, the electrokinetic ones are not and Newton's Third Law fails for electromagnetics.

However, as we have seen above, Newton's Second Law does not fail for electromagnetics. Newton worded it, "*The change of motion is proportional to the motive power impressed and is made in the direction of the right line [straight line] in which that force is impressed*" Today, we call his "motion" momentum or mv. So Newton is saying that the force acting on a body is equal to the rate of change of momentum of that body or as we have already noted: F = d(mv)/dt. This idea suggests what we call conservation of momentum. If there are no forces, then the total quantity of momentum can't change. This is a companion rule to what we've already observed where a zero Poynting vector (due perhaps to a zero electric or magnetic field) insures that energy stored in space must remain fixed there and cannot change.

Now consider if one has two concentric charged hoops there would indeed be some kind of action-reaction forces whereby if one charged hoop is accelerated, in one direction the other charged hoop would spin in the opposite direction. This is because torque on one hoop spins it creating an increasing current which produces an electrokinetic field at the second hoop which accelerates it and vice versa.[63] But if one hoop were a solenoid and not a cylinder with fixed charges and it is conductive wire with equal numbers of positive and negative charges, therefore, it can experience no force from the electrokinetic fields, which are tangential to it. Only the flow of currents in the solenoid can be affected.

Since the current represented by the rotating hoop also creates a magnetokinetic field we must examine that as well as a possible source of reactive torques on the solenoid. Notice that the current elements in the hoop are tangential to the current elements in the solenoid, which implies forces as found between parallel wires. However, the circular symmetry of the situation means that all these radial forces that may occur are balanced by radial forces on the other side of the hoop and being radial in any case can produce no torque upon the solenoid.

The end result of these considerations is that the charged hoop experiences forces, which result as a torque that rotates it while no reaction appears back upon the solenoid creating that torque. In short, Newton's Third law does not hold in this case. In fact Jefimenko derives the situation when the Third Law does not hold and in general it is in cases of

[63] Jefimenko, Oleg D., "Causality Electromagnetic Induction and Gravitation" ` 2nd Edition. Electret Scientific Co. Star City, WV, 2000, see example 3-3.6 sec 3-3, pp/ 56-57.

kinetic fields. He shows that give two charge distributions ρ_1 and ρ_2 where ρ_2 is either moving or time variable, the force on ρ_1 is given by: [64]

$$\int \rho_1 E_2 dv = -\int \rho_2 \ E_1 \ dv - \varepsilon_o \int E_1 \times \frac{\partial B_2}{\partial t} \ dv$$

Which shows balanced electrostatics forces but an unbalanced electrokinetic force.
A similar situation exists for magnetic forces from a changing current:[65]

$$\int J_1 \times B_2 \ dv = -\int J_2 \times B_1 dv + \int B_1 \times \frac{\partial D_2}{\partial t} \ dv$$

where it is again seen that action-reaction does not hold.

The failure of Newton's Third Law in electromagnetics is very interesting in that it suggests the theoretical possibility of a device from science fiction known as a "force glove". The idea is that this device is worn by the operator and can create large forces without any reaction on the person wearing the "glove". No planting of feet or straining "against" anything for footing is required to walk up and push over a building. And with a bit of imagination one can simply mount the "force glove" on the rear of a starship to achieve the mythical electromagnetic "impulse" drive to propel it through space. Likewise one could mount this glove under a craft to lift it as an "anti-gravity" device. These are interesting theoretical musings although, one must remember that this discussion is a classical one and relativity which could provide reaction forces is not considered.

Furthermore, one more remarkable feature was pointed out by Jefimenko:[66]

"*Thus, when both charge distributions are time variable or are in motion, the law of action and reaction, in general, does not hold: The two forces differ by the of the two integrals containing B_1 and B_2. However if the two integrals happen to be equal in magnitude but have opposite signs, they cancel each other, so that in this case the law of action and reaction does hold even when the two charges vary or move."*

"It is important to note that [the action-reaction equations] involve only the interaction, or mutual, momentum rather than the total momentum electromagnetic momentum of the systems under consideration. … A remarkable feature of these equations is that they involve only partial fields: E_1 and B_2 or E_2 and B_1. This means that even in a region of space where the total field $E = E_1 + E_2$ or $B = B_1 + B_2$ is zero, there still can be an exchange of electromagnetic and mechanical momentum."

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64 Jefimenko, Oleg D., "Causality Electromagnetic Induction and Gravitation" ` 2nd Edition. Electret Scientific Co. Star City, WV, 2000, p. 70.

65 Jefimenko, Oleg D., "Causality Electromagnetic Induction and Gravitation" ` 2nd Edition. Electret Scientific Co. Star City, WV, 2000, p. 74.

66 See Jefimenko "causality" p. 71 and p. 77.

Field Superposition

An example of this partial field phenomena is shown in Figure 10. below based on experiments suggested by Professor Hooper.[67] A setup after Faraday places two current-carrying wires equally distant from a sensing wire. Equal current is fed to the wire segments at right angles so as not to interfere with the actions. It is seen that these currents create magnetic fields equal and opposite at the sensing wire. Hence the total B field there is zero even if the currents are rising or falling. The electrokinetic E_k field still exists and for rising current is in a direction opposite to it and hence is the same for both sources when $I_1 = I_2$.

We have previously shown that a Lorentz E_L field is also generated by the causal electrokinetic term when there is motion. In this case we arrange the source wires on trolleys so they both move away in an identical manner where $V_1 = V_2$. This maintains the B field zero at the sensing wire even though each partial B field is slightly falling in magnitude, but the two relative motions are opposite creating a Lorentz E field in the same direction. So clearly as Jefmenko suggests above, the partial fields are superimposed and are still operating independently rather than being canceled to zero.

But failure of Newton's Third Law does not necessarily imply the failure of his second law! Momentum can remain conserved so long as one allows the concept that not only energy, but also momentum and angular momentum can be stored and transmitted in space as represented by fields. This is required because fields are the ONLY connection between charges and currents. Therefore all actions require fields, whatever those may actually be, to create actions between them.

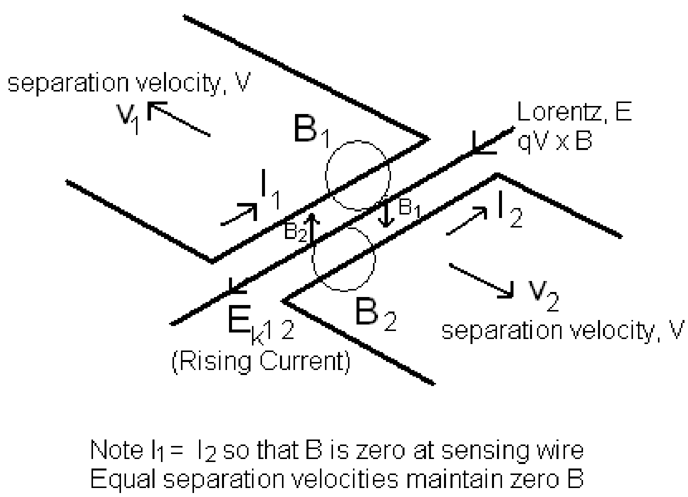

Figure 13. Partial Field Forces

[67] Hooper, W. J., "New Horizons in Electric, Magnetic and Gravitational Field Theory", Tesla Book Co., Calif. 1974, p82 ff.

Thus, our apparatus of Figure 13. not only demonstrates partial fields, but also the existence of force fields where a magnetic field is zero. This is interesting because that means the Poynting vector is zero at that location which says that there should be no energy flow in or out of the sensing wire, and that there is no momentum density stored there even if the sensing wire is replaced by a line charge and experiences an acceleration. While we've seen that energy stored in space must be calculated using the *total* fields, this seems to suggest that Poynting fields in space may operate with partial fields the way forces do.

Angular Momentum

We begin with Newton's Second Law. This law that states that force is equal to the rate of change of momentum can be written:

$$\vec{F} = \frac{d(\vec{p})}{dt} = \frac{d(m\vec{v})}{dt} = \vec{v}\,\frac{dm}{dt} + m\vec{a}$$

This shows that force can arise from either a rate of change of mass or a rate of change of velocity. The Law of Conservation of Momentum states that if no external forces act then the total system momentum equals a constant value. But our interest here involves rotating systems and angular momentum. If mass is constrained to move in a circular path it is defined as having an angular velocity, □, that is a vector in the direction of the axis of rotation with magnitude given by:

$$\omega = \frac{d\theta}{dt}$$

Which has a relationship to the tangential velocity at any point in the rotating mass given by the relationship:

$$\vec{\boldsymbol{v}}_t = \vec{\boldsymbol{\omega}} \times \vec{\boldsymbol{r}}$$

where **v** is the tangential velocity vector of the point, **r** is a radius vector from the axis of rotation to the point and □ is the vector angular velocity (in the direction of the axis of rotation). Angular momentum of a rotating (orbiting) particle is therefore defined as:

$$\vec{\boldsymbol{L}} = \vec{\boldsymbol{r}} \times \vec{\boldsymbol{p}}_t$$

Where **r** is the usual radius vector and $\mathbf{p_t}$ is the tangential linear momentum as defined by the tangential velocity expressed above. "Force" in rotating systems is called "torque" and is given by a law analogous to the Newton's expression:

$$\vec{\boldsymbol{\tau}} = \frac{d\,\vec{\boldsymbol{L}}}{dt} = \left(\frac{d\,\vec{\boldsymbol{r}}}{dt} \times \vec{\boldsymbol{p}}\right) + \left(\vec{\boldsymbol{r}} \times \frac{d\,\vec{\boldsymbol{p}}}{dt}\right) = \vec{\boldsymbol{r}} \times \frac{d\,(m\vec{\boldsymbol{v}})}{dt} = \frac{d\,\vec{\boldsymbol{L}}}{dt}$$

which states that torque acting on a system is equal to the time rate of change of its angular moment..

To complete our review, we now note that the rate of change of **ω** is called angular acceleration and is analogous to linear acceleration in Newton's Laws. This can be shown to lead to a rotational analog to the Newtonian formula **F** = m**a:**

$$\vec{\tau} = O\,\vec{\alpha}$$

Note I am using "O" here in place of the usual "I" for rotational inertia because eventually we are going to be talking about fields and currents and we do not wish to confuse rotational inertia with electric current. "O" was picked because the letter shape implies a circular path.

It is also interesting to note that rotational inertia, O, unlike it's linear analog, mass, has a value that depends on more than just the amount material and its density. Rotational inertia depends upon the geometry of the distribution of matter and hence can vary depending upon which axis of rotation it chosen for any given shape. In this way it is analogous to inductance where the mechanical inertia of water in a hose depends only on the length and diameter of the hose and not upon it's shape, while the inductance of a wire, depends not only on the length and diameter of the wire but also upon it's configuration in space. For example a coiled wire has a much greater self-inductance than the same wire laid out in a straight line.

Since Angular momentum of a rotating (orbiting) particle was defined above as:

$$\vec{L} = \vec{r} \times \vec{p}_t$$

One notes that when a volume of space contains a certain amount of (tangential) linear momentum, it is clear that the vector from the axis of rotation to that stored momentum must be specified. While the actual trajectory may depend upon the choice of physical pivot, but once the trajectory is fixed, it is important to note that the value of angular momentum calculated from any arbitrary pivot point if no torques are present is always constant and does not depend upon the choice of pivot. In short, we would say angular momentum is conserved. Hence if a charged particle is injected into a uniform static **B** field where it continues to orbit with a certain angular momentum, it is clear that to keep the total angular momentum zero and equal amount of angular momentum must be stored in space. When the **B** field is turned off then that angular momentum comes back out of the field to cancel the angular momentum of the previous orbit.

At this point we would like to take note of an important fact: energy can go in and out of a system while angular momentum is conserved! Consider a spinning skater or a person sitting in a desk chair rotating with weights held at arm's length. When the arms or weights are pulled in the angular velocity of the spinner greatly increases because the geometry and hence O of the system has changed. If you calculate the energy of both conditions, you find that the kinetic energy of the person has greatly increased as they spin faster while angular momentum was conserved and remains constant. How can that be? It turns out that *energy* was added to the system through the *work* the spinner did pulling the arms or weights in! Hence *energy* was added (or can be taken out again by extending arms) but angular momentum was *not* changed which was shown by the change in angular velocity. Angular momentum is unchanged because no torque was added or removed from the system even though energy flowed in and out.

An example of how energy can be stored in space as angular momentum and later extracted from space from that angular momentum while the total angular momentum remains zero is seen in Fig.14.

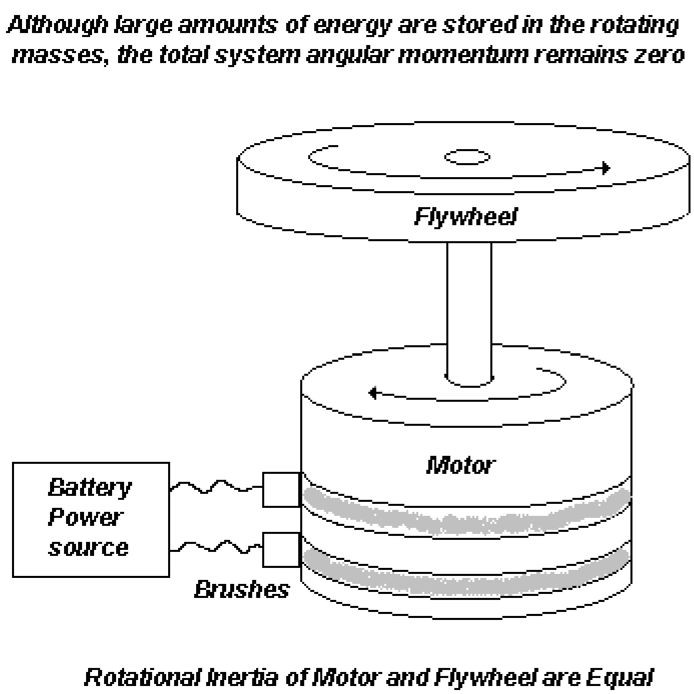

Figure 14. Energy from battery can be stored in space using angular momentum with this apparatus and then later can be extracted to charge the battery.

In this case energy spins the motor and flywheel in opposite directions so that total angular momentum is always zero. Nevertheless the energy is stored in the rotating parts and can be extracted from them. Also note that the rotational inertia of the motor and flywheel need not be equal, but the angular velocity of the two parts will no longer be equal and opposite since the angular velocities will adjust themselves to maintain total angular momentum equal to zero.

We have already noted that electric fields are force fields and if we examine the forces of our three types of electric fields on say a circular ring of charge, we find that the Lorentz E field requires an additional velocity and a B field which would be zero in free empty space, but a uniform static B field will bend the trajectory of a moving charged particle into a circular orbit by reason of the Lorentz E field. And while an electrostatic E field does produce a force, the torque experienced by a ring of charge and similar objects requires a non-zero closed line integral of the electric field, and a conservative electrostatic field always gives a zero value to these integrals. And Jefimenko notes[68] "…*only the electrokinetic field gives a non-vanishing contribution to such integrals* (*The first term of* [causal equations for an E field] *being a function of r in the direction of r, has zero curl and therefore cannot contribute to closed line integrals*)". For this same reason, induction of currents in closed secondary loops can *only* be due to Electrokinetic E fields.

Angular Momentum and the Magnetic Vector Potential

We have already noted that only an electrokinetic E field can produce torque on a charge distribution. Therefore only an electrokinetic field can result in angular momentum transferred to charges. But conservation of angular momentum assures us that if we start with zero angular momentum before we turn the current on, it must remain zero as the charges are placed into motion. Therefore, it is clear that if an electrokinetic field accelerates a charged hoop, for example, the angular momentum gained by that hoop must be exactly balanced by another equal, but opposite, batch of angular momentum that is somehow stored in space represented by fields. This will be examined in more detail below.

Finally when there is a circular electrokinetic field action on a charge distribution restricted to circular motion (such as the charged rings we will examine) the angular momentum changes. When the charge distribution and electrokinetic field are of circular symmetry, the change in angular momentum acquired by the charge distribution is given by:[69]

$$\Delta \vec{\boldsymbol{L}} = \iint \vec{\boldsymbol{r}} \times \rho \vec{\boldsymbol{E}}_{\boldsymbol{K}} \, dq \, dt = - \int \vec{\boldsymbol{r}} \times \rho \vec{\boldsymbol{A}} \, dq$$

as

$$\boldsymbol{E}_{\boldsymbol{k}} = -\frac{\partial \boldsymbol{A}}{\partial t}$$

As pointed out above, the retarded magnetic vector potential must be used for an exact solution, but for our instructive learning purposes, we can always assume that the driving currents are of such a form as to create a linearly rising or falling E_K such that our simpler unretarded equations are nevertheless exact. And as noted above since the axis of rotation depends upon the geometry of the apparatus, the quantity of angular momentum is not absolute but depends on the arbitrary choice of that geometry.

At this point to help us understand these electromagnetic relationships we are going to examine something called "Feynman's Paradox". This was a puzzle Feynman put to his students in his famous lectures to get them to think about the idea of fields ("empty" space) carrying angular momentum.

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68 Jefimenko, Oleg D., "Causality, Electromagnetic Induction and Gravitation", Op cit. Appendix 5, p.44..

69 See Jefimenko, "Causality Electromagnetic Induction and Gravitation ", p 43

Feynman Paradox

Feynman describes the apparatus and the paradox: [70]

"Imagine that we construct a device like that shown in Fig. 17-5. There is a thin circular plastic disk supported on a concentric shaft with excellent bearings that is quite free to rotate. On the disc is a coil of wire in the form of a short solenoid concentric with axis of rotation. This solenoid carries a steady current I ..."

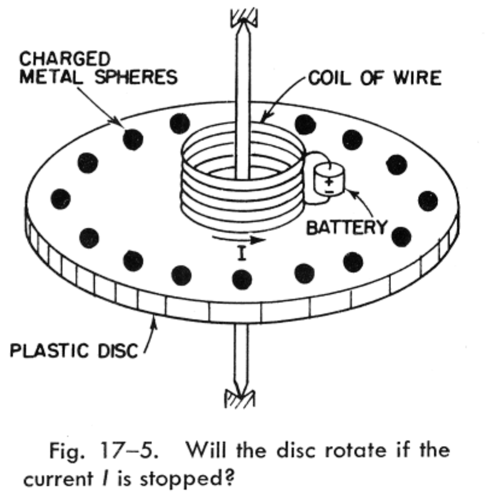

Figure 15. Feynman Paradox

Then he even "improves" the apparatus making the coil super-conducting so there is no battery and the current will stop when the coil heats up. He then says:

"So long as the current continued, there was a magnetic flux through the solenoid more or less parallel to the axis. When the current is interrupted this flux must go to zero. There will, therefore, be an electric field induced, which will circulate around in circles centered at the axis. The charged spheres at the perimeter of the disk all experience an electric field tangential to the disk. This electric force is in the same sense for all the charges and so will result in a net torque on the disk. From these arguments we would expect that as the current in the solenoid disappears, the disk would begin to rotate. If we knew the moment of inertia of the disk, the current in the solenoid, and the charges on the small spheres, we could compute the angular velocity."

Feynman then describes the second way of looking at this problem:

"But we could also make a different argument. Using the principle of the conservation of angular momentum, we could say that the angular momentum of the disk with all its equipment is initially zero, and so the angular momentum of the assembly should remain

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[70] Feynman, Leighton, Sands, "The Feynman Lectures on Physics", Addison-Wesley Co. Palo Alto, 1963, Section 17-4 Volume II.

zero. There should be no rotation when the current is stopped. Which argument is correct? Will the disk rotate or not? We will leave this question for you to think about."

Before we discuss Feynman's "explanation" of this paradox we will actually do the calculation he suggests, though with several changes in his apparatus to make the calculations simpler.

The Modified Feynman Apparatus

Our modified apparatus to simplify the Feynman paradox will consist of a cylinder of mass m, radius r_o, and charge q located inside a long solenoid of radius, R. The fact that the fields about a long solenoid are well known plus the fact that Jefimenko has already partly worked out this problem[71] makes the task easy. We will also be calculating the rotation of the cylinder when the current is applied rather than the reverse where the current is stopped as Feynman did, as well as studying a number of other conditions to try to gain some insight into energy, momentum and fields.

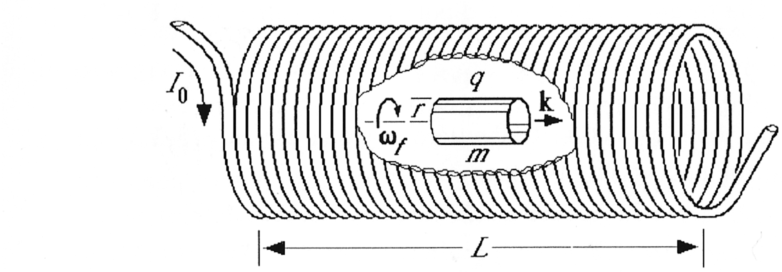

Fig. 3.9 A charged cylinder placed inside a solenoid rotates when the current is switched on. If the charge of the cylinder is positive, the rotation is against the direction of the current.

Figure 16. Modified Feynman Experiment.

Because of the redundancy of electromagnetic relationships, there are a number of different ways the above situation can be calculated all yielding the "correct" answer. As can be seen above, Feynman suggested using Faraday's law where the change in flux through the surface area of a closed loop creates an emf in that loop which can be expressed in our case as an electric field, E times the distance around the hoop, $2\pi r_o$, where r_o is the radius of the circular hoop (cylinder).

The magnetic field inside a long solenoid of radius R made up of N turns per unit length L for a given current I is well known, is parallel to the axis of the coil and given by:

$$B = \left(\frac{N}{L}\right)\mu_o I$$

[71] See problem 3-3.4, p.54 in Jefimenko's book, "Causality, Electromagnetic Induction and Gravitation",

Outside the long solenoid the length has pushed that magnetic field away to a great distance so near the coil there is essentially no magnetic field in the region just outside the coil near the cylinder. Conceptually, employing a toroid rather than a solenoid easily solves the problem of end effects and the returning fields outside a long solenoid. However, here we shall use a solenoid to simplify calculations. Note that the magnetic field does not depend upon the diameter of the coil. Our first consideration will be the basic operation of the solenoid coil itself, which is obviously an inductor.

Fields, Self-inductance and Energy Storage of a Long Solenoid

A long coil of wire in the form of a solenoid or toroid is common form of an inductor. A current in the coil produces a magnetic field inside the coil but not just outside the coil. In the case of a toroid the magnetic field is completely confined to the interior of the coil. The magnetic field is parallel to the axis of the coil. We have observed that a magnetic field stores energy. Thus, a current carrying coil stores energy in the space of it's interior. In the case of a toroid no energy is stored outside the coil because the magnetic field is always zero there.

If the current in the coil is rising or falling, an Electrokinetic E field is created at the coil. This field is either parallel or anti-parallel to the current and thus can either assist or resist the flow of current in the coil. This field also exists inside and outside the coil. Inside the coil the E field is in circles about the axis of the coil. If one crosses that tangent E field into the magnetic B field a Poynting vector is created. It is observed that this Poynting vector is radial and points into the interior of the coil when the current is rising and out of the coil interior when the current is falling. This shows that the inductive stored energy flows into and out of the interior space of the coil as current rises or falls.

The Problem of Self-inductance

If we wish to actually calculate the effects described in the last section, there are problems that arise whenever an attempt is made to calculate self-inductance from electrokinetic action (as happens for example in the Neumann equation). The problem is that the causal electrokinetic fields vary as 1/distance from the source currents. If one is calculating mutual inductance where some finite distance is involved, this is no problem. However, for self-inductance (or in the above case, generation of an opposing field that reduces the forces creating rotation) as the integration distance approaches the source current we have an undefined division by zero situation. This is why calculation of self-inductance is always done with tricks and hand-waving or redundancy. This is also why the self-inductance of a wire tending to a zero diameter tends to infinity. One can, of course simply trim all distance values smaller than some arbitrary amount in the calculations, but that really does not address the true problem. We'd like to point out here that the fundamental problem is that physicists generally do not understand cybernetics, which is to say feedback systems. Engineers usually do have acquaintance with them, but generally have not applied the ideas to these electromagnetic situations. The key point here is that opposing electrokinetic fields as in our above rotating cylinder can never get greater than primary electric field driving the current! Infinite fields at zero distances simply never occur even theoretically. That is the "feedback" situation.

Once this situation is understood it can be shown that self-inductance not only is a function of only the geometry of the conductor, but also that the inductance is distributed in varying amounts over the conductor. For example the corners of a rectangular bus bar have lower inductance than its center. This means that more current flows in the edges of a bus bar in response to a step potential than in the center of it. We will not be discussing the "feedback" problem of self-inductance here as this could be the topic of an entire paper, but we do wish to indicate that it can be dealt with and things like the distribution of inductance over the geometry of conductors can indeed be calculated in spite of the problem of "infinite" mutual coupling at small distances that renders the Neumann equation unusable for self-inductance calculations. Our above momentum distributions between kinetic and electromagnetic can also be calculated from the same principles.

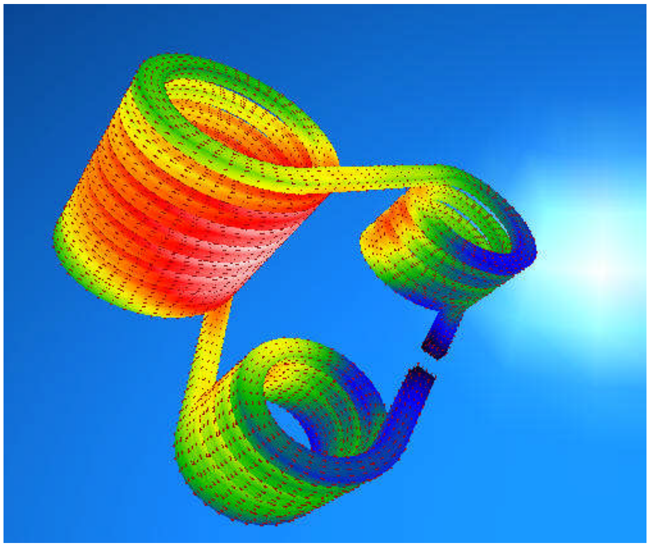

Figure 17. Example of Inductance Distribution over a Conductor. (Distinti Thesis)[72]

The distribution of inductance over a conductor is observed in Fig. 14. Note that isolated conductor elements have lower inductance (blue) while elements with many nearby supporting current elements supplying electrokinetic fields have much higher inductance (red). The salient point here being that each current element in a conductor of some given geometry generates electrokinetic E_K fields about itself that penetrate all other current elements in the configuration altering their total fields driving the currents in them. When an inductor is made from a long thin conductor (wire) as above, then an additional condition is imposed where the total current in each element must be the same. The voltage across each element however will not be the same and reflects the inductance at that point as demonstrated in Figure 17 above.

[72] See website: http://www.newelectromagnetism.com/ ; Click on "NE Grad Thesis".

Pointing Flow when a Charged Cylinder is Placed Inside a Solenoid

Having examined energy storage inside a coil, we now need to examine the situation when a charged cylinder is placed inside the coil while it is carrying a current. This situation is diagrammed in Figure 15. below.

In this situation the Electrostatic field of the charged cylinder is seen to be radial while the magnetic field of the coil remains parallel to its axis. As seen above this creates a Poynting vector that represents a circulation of energy inside the coil about its axis, rather than just a storage of stress. Later we shall see that Feynman relates this circulating energy flow to the storage of angular momentum within the coil. Again note that since the B field is zero outside a toroid all the electromagnetic angular momentum (circulating energy) as well as the inductive magnetic energy is stored within the interior of the coil.

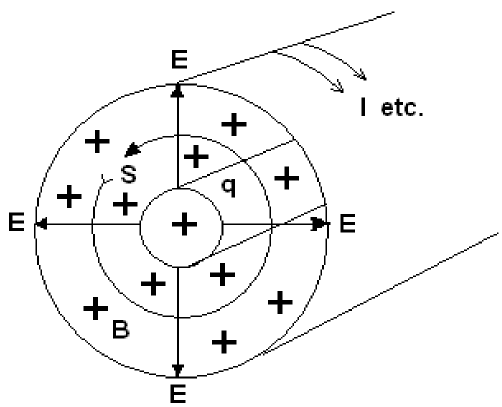

Figure 18. Circulating Energy Inside a Long Solenoid.

The Feynman Calculation: Faraday's Law

To continue with our calculations with respect to our modified apparatus we observe that Faraday's Law is given by:

$$emf = -\frac{d\Phi}{dt}$$

When the charged cylinder is placed inside the solenoid coil, **B** is uniform over the cylinder and the flux is determined by

$$\Phi = \int_{AREA} B\, ds = \pi\, r_o^2\; B = \left(\frac{N}{L}\right)\mu_o\, \pi\, r_o^2\; I$$

Where the “area” is the surface defined by the end of the cylinder. Therefore

$$emf = E_K\;(2\pi\, r_o) = -\left(\pi\, r_o^2\right)\frac{dB}{dt} = -\left(\pi\, r_o^2\right)\left(\frac{N}{L}\right)\mu_o\,\frac{dI}{dt}$$

Thus we find a tangential electric field accelerating the cylinder that is equal to:

$$E_{TANGENTIAL} = -\left(\frac{N}{L}\right)\frac{\mu_o\ r_o}{2}\frac{dI}{dt}\,\overline{u_\theta}$$

For cylinders of radius r_o *inside* the solenoid coil.

Where u_θ is a unit vector in the theta (tangential) direction. Note that the field does not depend upon the diameter of the solenoid but does depend on the diameter of the cylinder. The negative sign represents Lenz's law, which says that the magnetic field produced by the current represented by the rotating cylinder must be created in opposition to the increasing magnetic field of the solenoid. Since dI/dt is positive for a rising current and negative for a falling one, the tangential $\mathbf{E_K}$ field is directed opposite to the current for an increasing current and in the direction of the current for a decreasing current. The tangential force at the positively charged cylinder is given as usual by the product of the tangential $\mathbf{E_K}$ field value and the total charge (since the field does not vary around the hoop).Obviously this tangential field will apply torque to the cylinder, rotating it and giving it mechanical angular momentum or removing angular momentum from it. Later it will also be observed that such rotation produces a current and resultant additional magnetic field.

Another case of later interest will be when the cylinder is larger and located *outside* the solenoid. In this case the total flux is fixed and given by the above relation where R replaces r_o in the formula for the area covered with uniform magnetic field.

$$\Phi = \int_{AREA} B\ ds = \pi\ R^2\ B = \left(\frac{N}{L}\right)\mu_o\ \pi\ R^2\ I$$

Since the emf equals the integral of $\mathbf{E_k} \bullet \mathbf{dl}$ around the cylinder we find:

$$emf = E_K\ (2\pi\ r_o) = -\left(\pi\ R^2\right)\frac{dB}{dt} = -\left(\pi\ R^2\right)\left(\frac{N}{L}\right)\mu_o\ \frac{dI}{dt}$$

Which produces a tangential $\mathbf{E_k}$ field outside the solenoid given by:

$$E_{TANGENTIAL} = -\left(\frac{N}{L}\right)\frac{\mu_o\ R^2}{2\ r_o}\frac{dI}{dt}\,\overline{u_\theta}$$

For cylinders of radius r_o *outside* the solenoid coil of radius R of N turns per unit length.

Note that for cylinders of radius R either just inside or just outside the coil, the tangential force and hence torque will be the essentially the same in either case.

The Feynman Calculation: Causal Maxwell's Equations

A second, though more complex method of solution for the tangential field would be to use the causal expression for the Electrokinetic E_K field. Here an integral is made over then entire current source, which is the current-carrying helix of the solenoid. We won't do that here, but it is certainly possible mathematically. However, we will point out that the negative sign of that term along with the derivative of the current density produces a field direction opposite to the current vector for increasing current (positive derivative).

This solution is remarkable in that it involves no magnetic fields whatsoever. The calculation is simply between the changing currents in space and the electrokinetic fields created by them.

The Feynman Calculation: Magnetic Vector Potential

The final redundant method which will be employed here, is based upon the relationship:

$$\vec{E}_K = -\frac{\partial \vec{A}}{\partial t}$$

discussed above.

The magnetic vector potential inside and outside a long solenoid is well known and is a solenoidal field. This means that the A fields is in closed loops inside and outside the coil. In cylindrical coordinates it has no r or z components. A is always in the theta tangential direction, but has no variation in the theta or tangential direction as well as no variation in the z direction. This is important because the magnetic vector potential A is related to the magnetic field B by the equation:

$$\vec{B} = \nabla \times \vec{A}$$

In cylindrical coordinates the curl of a vector A is given by:

$$\nabla \times \vec{A} = \vec{a}_r \left(\frac{1}{r} \frac{\partial A_z}{\partial \phi} - \frac{\partial A_\phi}{\partial z} \right) + \vec{a}_\phi \left(\frac{\partial A_r}{\partial z} - \frac{\partial A_z}{\partial r} \right) + \vec{a}_z \left(\frac{1}{r} \frac{\partial r A_\phi}{\partial r} - \frac{1}{r} \frac{\partial A_r}{\partial \phi} \right)$$

Which by the above conditions reduces to the simple value for the magnetic field B:

$$\vec{B} = \nabla \times \vec{A} = \vec{a}_z \frac{1}{r} \frac{d \ r A_\phi}{dr}$$

The magnetic vector potential inside a long solenoid depends on distance from the axis and is well known and given by:[73]

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[73] See Jefimenko, :"Electricity and Magnetism", p383, problem 11.2.

$$\vec{A}_\varphi = \mu_o \left(\frac{N}{L}\right) \frac{r}{2} I \, \vec{u}_\theta$$

Which by the above equation gives a B field in the axis (z) direction identical to our previous calculation.

$$B_z = \frac{1}{r} \frac{\mu_o}{2} \left(\frac{N}{L}\right) I \frac{d r^2}{dr} = \left(\frac{N}{L}\right) \mu_o I$$

Since

$$\vec{\boldsymbol{E}}_{\boldsymbol{K}} = -\frac{\partial \vec{\boldsymbol{A}}}{\partial t}$$

We find

$$E_{TANGENTIAL} = -\left(\frac{N}{L}\right) \frac{\mu_o \, r_o}{2} \frac{dI}{dt} \overline{u_\theta}$$

For cylinders of radius r_o *inside* the solenoid coil and is identical to that found with Faraday's Law.

Outside the solenoid the magnetic field is zero, which might lead to a conclusion that the magnetic vector potential is zero but that is not the case. A and B are also related by the equation:[74]

$$\oint \vec{\boldsymbol{A}} \bullet d \, \vec{\boldsymbol{l}} = \int \vec{\boldsymbol{B}} \bullet d \, \vec{\boldsymbol{S}} = \Phi$$

And since A_φ is constant with respect to z and angle, θ, and the flux for cylinders outside the coil is also a constant and was calculated above, we obtain:

$$A_\varphi 2 \pi r_o = \left(\frac{N}{L}\right) \mu_o \pi R^2 I$$

Which gives a value for A_φ of

$$A_\phi = \left(\frac{N}{L}\right) \frac{\mu_o}{2} \frac{R^2}{r_o} I$$

Which taking the time derivative gives the value of E_k at the cylinder radius, r_o, of

$$E_{TANGENTIAL} = -\left(\frac{N}{L}\right) \frac{\mu_o \, R^2}{2 \, r_o} \frac{dI}{dt} \overline{u_\theta}$$

Which is the same as was found with Faraday's Law and if this value of A_ϕ is put into our expression above in cylindrical coordinates for B, it is seen that the radial distance, r_o, in

74 See Jefimenko, :"Electricity and Magnetism", p366

the denominator of A cancels the radial distance, r, in the numerator inside the derivative leaving us with the derivative of a constant which is zero. Hence B outside the solenoid is zero, but the vector magnetic potential (and tangential force on the cylinder) falls off as inverse distance from the axis of the solenoid.

Attempt to Calculate Angular Momentum of the Charged Cylinder

Our interest here involves angular momentum especially that acquired by and given up from the rotating cylinder which for now we have placed inside the solenoid. Therefore we now employ the relationship for the change in momentum ΔP acquired by the cylinder which is valid because $\mathbf{E_K}$ and **A** are constant values over the cylinder.

Hence

$$\Delta\vec{P} = -\left(\frac{N}{L}\right)\frac{q\,\mu_o\,r_o}{2}\,\Delta I = -q\,\Delta\vec{A}$$

where

$$\Delta\vec{P} = m\,\vec{v}_{Tangential} = m\vec{\omega}\times\vec{r} = -q\Delta\vec{A}$$

Therefore since r and ω are at right angles, the angular velocity attained by the cylinder as a result of current in the solenoid rising from zero to the value I is given by:

$$\vec{\omega} = -\frac{q\Delta\vec{A}}{mr_o} = -\left(\frac{N}{L}\right)\frac{q\,\mu_o}{2\mathrm{m}}\,\Delta I\,\vec{k}$$

Where **k** is a unit vector in the direction of the solenoid axis according to the right hand current rule. Which then raises the question of how much angular momentum has been imparted to the cylinder by the electrokinetic field? Since the moment of inertia, O, for a hoop spinning about its axis equals mr_o^2 and angular momentum, L, equals Oω we have the result:

$$\Delta L = -\left(\frac{N}{L}\right)\frac{q\mu_o\,r_o^2}{2}\,\Delta I\,\bar{k}$$

that agrees with our previous expression:

$$\Delta\vec{\boldsymbol{L}} = \iint \vec{\boldsymbol{r}}\times\vec{\boldsymbol{E}}_{\boldsymbol{K}}\,dq\,dt = -\int \vec{\boldsymbol{r}}\times\vec{\mathbf{A}}\,dq$$

Since **r** is at right angles to **A** and the integral of dq over the hoop gives q or

$$\Delta\vec{\boldsymbol{L}} = -q\,r_o\,\Delta A\,\vec{\boldsymbol{k}} = -q\,r_o\,\mu_o\left(\frac{N}{L}\right)\frac{r_o}{2}\,I\,\vec{\boldsymbol{k}} = -\left(\frac{N}{L}\right)\frac{q\,\mu_o\,r_o^2}{2}\,I\,\vec{\boldsymbol{k}}$$

Finally since torque is given by

$$\tau = \frac{dL}{dt} = F \times r_o = -qE_K r_o \bar{z}$$

for a constant moment of inertia, we see that the torque applied to the hoop is given at any moment by the derivative of the angular momentum. If the angular momentum is a linear ramp then the torque is constant and given by the slope of that rise.

Energy and Angular Momentum in Space

Therefore, we observe that from our apparatus with a charged cylinder inside the coil, which has no energy or angular momentum, in applying a current to the coil we rotate the cylinder creating a certain amount of mechanical angular momentum. There is also obviously a certain amount of mechanical energy stored in the rotating cylinder as well. This is in addition to the magnetic energy stored in the magnetic field of the coil due to its inductance.

From the above considerations we surmise that if conservation of momentum is indeed true, then somehow there must be "missing" angular momentum in the Feynman apparatus that is keeping the total angular momentum zero at all times. The suggestion Feynman makes is that this momentum is stored in fields. And previously it was shown how the Poynting vector represents a momentum density in space.

We start this discussion with an examination of the explanation Feynman gave in his Lectures. Feynman examines the case of a fixed magnet next to a fixed charge.[75] Note that the permanent magnet and a coil of wire carrying a DC current are equivalent in this case. The situation is shown in Feynman's figure 27-6 [Fig.16]. As usual, the static E field from the charge is totally radial and the magnet produces the flux lines typical of a short solenoid or permanent magnet. Both fields are totally static and unchanging.

Note that all the radial field vectors of the static E_S field when used in a cross product with the looping flux lines of the magnetic field, produces a Poynting energy flow in a circular pattern about the central plane of the magnet and disk. Most energy appears to be stored in that plane with little stored directly above or below the magnet where E_S and B are nearly in the same direction. Energy flow forced into a circular path represents angular momentum, which in this case appears to be angular momentum stored in the combination of two static fields. Note that even though we know energy can be stored in either a magnetic or electrostatic field by itself, there is no *circulating* energy if either one is zero as it drives the Poynting vector to zero.

[75] Feynman, op. Cit. Lectures, vol II, 27-5

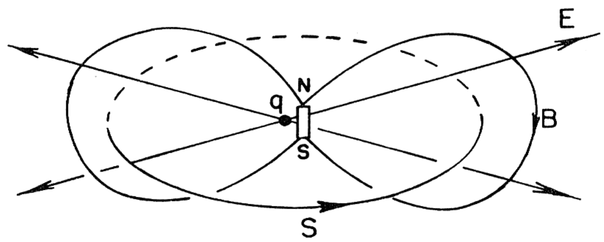

Fig. 27–6. A charge and a magnet produce a Poynting vector that circulates in closed loops.

Figure 19. Feynman's Circulating Poynting Energy

Feynman explains the paradox as follows at the end of Section 27:

"Do you remember the paradox we described in Section 17-4 about a solenoid and some charges mounted on a disc? It seemed that when the current turned off, the whole disc should start to turn. The puzzle was: Where did the angular momentum come from? The answer is that if you have a magnetic field and some charges, there will be some angular momentum in the field. It must have been put there when the field was built up. When the field is turned off, the angular momentum is given back. So the disc in the paradox would start rotating. This mystic circulating flow of energy, which at first seemed so ridiculous, is absolutely necessary. There is really a momentum flow. It is needed to maintain the conservation of angular momentum in the whole world."

"Hidden" Angular Momentum

And all of the above is well and good but it really isn't a calculation of field momentum. For that we will take a closer look at our little experiment. We know that the tangential forces on our charged cylinder depend upon the changing current in the coil and upon the charges on the hoop. If the charge is zero then there is no torque and no momentum. A rising or falling current does not spin an uncharged cylinder. If we double the charge we've seen that we double the mechanical momentum the hoop acquires and also doubles the angular momentum stored in space.

Furthermore, in Feynman's experiment he started with a current in the coil but the disk not rotating as initial conditions. So our premise is that when we increase the coil current to a value the cylinder attains a certain value of mechanical momentum and an equal but opposite momentum gets stored somehow in space represented by fields keeping total angular momentum zero. If one were now to reach in and absorb that mechanical angular momentum into "the world" by stopping the hoop rotation, presumably the stored field momentum would remain. And indeed one finds that if the hoop is not rotating, once the current stops it begins to rotate. Since this system can be totally isolated (say we use a superconducting coil that heats up as Feynman suggests), the angular momentum going into mechanical rotation and found at the end when there are no fields, in this case somehow must be coming out of our theoretical angular momentum stored in space. This conserves momentum and keeps it constant with the final mechanical momentum value being equal to the amount of "hidden" electromagnetic momentum stored in space.

Or we can perform another experiment. Suppose we have raised the current and the cylinder is spinning. But now we bring in a wire and ground the cylinder removing all the charge. We suggest that this can be done such that it keeps spinning[76] so we have removed the angular momentum stored in fields but have not removed the mechanical momentum. Feynman suggests that a circulating Poynting field stores momentum and without charge there cannot be a non-zero Poynting vector as both E and B are necessary. But interestingly when we stop the current in the coil the cylinder does not stop because without charge it experiences no torque. And we can surmise that in this case we removed the momentum from space while leaving the mechanical momentum in place. In other words, it is the opposite case of our previous experiment and again angular momentum is conserved, as the mechanical momentum does not change when the fields change.

The EM fields of the Driving Solenoid

To understand how electrokinetic fields work, we will start with an examination of the fields of the driving solenoid. We have assumed a coil of N turns per unit length driven with a current that begins at zero, rises in a linear ramp to a value I at some time t_o later. The current is assumed driven by an electronic current source that controls the current by manipulating the applied voltage. Thus, driving impedance questions can be ignored.

Note that when the power supply is switched on it attempts to get current flowing by applying a potential to the coil. But by the Jefimenko causal equations for **E** and **H** fields, an electrokinetic $\mathbf{E_K}$ field is setup about each current element **dl** in the wire. That field is in opposition to the applied field that is attempting to drive a current through the wire. Furthermore, the magnitude of the opposing field is determined by the time rate of change of the current in the wire. Therefore, the faster the current tries to flow the more the electrokinetic field cancels the propelling applied field generated by the voltage difference applied to opposite ends of the coil. Looking at any given current element dl, one can see that the electrokinetic $\mathbf{E_K}$ field observed there is the sum of all the electrokinetic fields of all current elements in the coil. But it can also be calculated from the negative of the time rate of change of magnetic vector potential at that point.
And furthermore, a problem is noted where one begins to integrate fields from nearby elements where the Electrokinetic field suddenly seems to be rising to infinite values as one draws near the element in question due to the distance term in the denominator. Of course this isn't true, as this is a feedback problem where the induced electrokinetic fields

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[76] Suppose we bring in a grounded conductive tube the diameter of the cylinder to one of it's ends and get it so close that sparks jump to it. All currents are parallel to the magnetic field so that I **dl** x **B** is zero and the current represented by the leaving charge produces no torque or forces on the rotating cylinder. One could also charge the stationary cylinder this way after the coil current has been established putting momentum into space without rotating the hoop establishing Feynman initial conditions.

can never rise to more than to exactly cancel the applied field to zero. They don't create energy. They merely cancel applied energy. The net result of all this can be seen in the old electrical engineering rule that current in an inductor or voltage in a capacitor cannot change instantaneously. Clearly the electrokinetic field is the source of self and mutual inductance. And is the basis of the well-known relationship for inductors as circuit elements:

$$V = L\,\frac{dI}{dt}$$

where L is termed the self-inductance and is only a function of the geometry of the conductors in space. Furthermore any current in the coil creates a Biot-Savart **H** field which we have shown stores energy. We have already seen that the energy stored in a magnetic field is related to the integral of that field squared integrated over all space. Furthermore, since we have seen that the B field in a solenoid is proportional to I, it should come as no surprise that the energy stored in the inductance above can be shown proportional to I squared:

$$U = \frac{1}{2} L\, I^2$$

Thus it is easily seen that the effects of changing current creating electrokinetic fields are reflected back into the source supplying the driving current and appear as the self-inductance of that particular conductor geometry (solenoid coil in this case).

The EM Fields of the Rotating Cylinder

But we need to take a closer look because things are not quite as simple as they seem! Our original calculation of final cylinder velocity is only valid if the cylinder does not significantly rotate (has a very large mass) or is of a very small diameter. What has been missed is that the rotating charged cylinder represents a current and that current is varying. This creates not only a secondary magnetic field and a secondary magnetic vector potential vector, but a secondary electrokinetic E field as well.

Since our situation is actually more complex than we assumed, let us examine the case where rather than a charged hoop with fixed charges, a secondary conductive loop is placed inside or outside our solenoid coil. It should be clear that the electrokinetic E field generated by the driving current changes *also* appears in this conductive loop driving a current. It takes very little electric field to drive considerable current in good conductors. That current for an increasing drive current in the solenoid is in the opposite direction to that drive current (as is the electrokinetic field generated by it). Therefore we can easily see that this new current also generates a Biot-Savart magnetic field that is opposite to that generated in the drive solenoid by it's own current. Hence the secondary loop cancels part of the solenoid magnetic field and hence is able to reduce the energy stored in the total magnetic field. Furthermore, since there is a current rising in the secondary loop, it also creates an electrokinetic E field that partially cancels the electrokinetic E_K field generated by the solenoid drive current. Thus, this secondary electrokinetic field is seen to cancel part of the Electrokinetic field opposing current change in the driving solenoid.

The bottom line as is known, is that a shorted turn can greatly lower the self-inductance of a transformer. For this reason a much lower voltage produces the same current changes and thus less energy is drawn from the power source which makes sense because with lower self-inductance, less energy is being stored in space for a given current.

But in our case instead of a looped conductor we have a charged cylinder in or about the solenoid. In many ways it is the same as the above situation, except that the cylinder has mass and is capable of storing kinetic energy and momentum. In passing we can note a transformer action as well, given that the cylinder when rotating represents what is termed a “current sheet” and represents one turn over it's length compared to some number of turns in the solenoid over the same distance. Since the charged cylinder is increasing in angular velocity under the torque produced by the electrokinetic field of the driving coil, this represents an increasing secondary current that also produces a secondary electrokinetic field. This secondary electrokinetic field is in a direction to not only cancels part of the primary electrokinetic field of the drive coil reducing it's inductance, but also opposes the rotation of the charged cylinder acting as a kind of “inductance” effect reducing the torque on the charged cylinder as it picks up angular velocity.

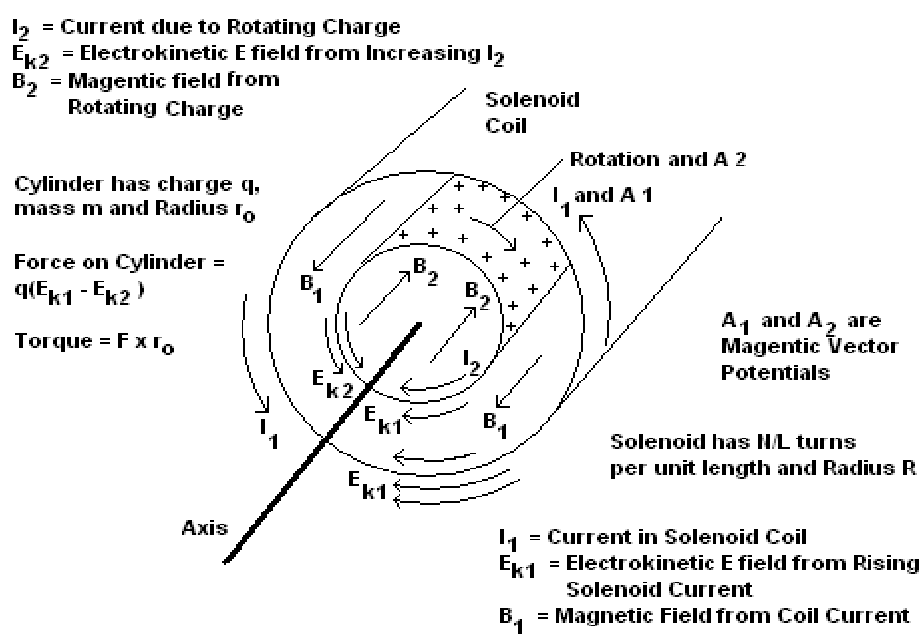

Figure 20. Fields for New Feynman Apparatus

Our field situation with the rotating charged cylinder is diagrammed in Figure 20. and from the figure an outline of the operation of our apparatus can be surmised. We begin with no rotation of the charged cylinder and no current in the solenoid coil. Current in the solenoid rises linearly from zero to a final value I. This current creates a Biot-Savart field, B_1 inside the solenoid. That B field not only stores energy which appears as the inductance of the coil, but also is associated with a magnetic vector potential A_1 in the direction of the current both inside and outside the solenoid. The time rate of change of the solenoid current, which is rising or positive in this case, creates an Electrokinetic E_{k1} field in space about the solenoid that can be calculated from the time rate of change of the A_1 field. E_{k1} is oppositely directed to the driving current. Hence this inductance requires higher voltages to drive the same current rise.

Since the solenoid driving current is a linear ramp, E_{k1} is a square pulse which suddenly rises to a fixed value. This field acts on the charges of the cylinder to produce a torque T that gives it angular acceleration attempting to rotate it opposite to the flow of current in the coil and $A_{1.}$. But the cylinder having mass cannot move instantaneously. Therefore the cylinder begins stationary and then rotates with increasing velocity as the torque is applied to it attaining a final angular velocity as the current ceases to rise when it reaches the value I.

Were this the end of the story, we have already calculated these values. But since the rotation of the charge on the accelerating cylinder represents a current loop I_2, there are additional effects. The current represented by the rotation of the cylinder also creates a magnetic field, the B_2 field, which is opposed to the previous B_1 field. Therefore since part of that field is canceled, the inductive stored energy is reduced, lowering the apparent inductance of the drive coil. Furthermore, since the cylinder rotation is accelerating, the apparent current is increasing giving rise to a secondary electrokinetic field about the cylinder opposite to the direction of its rotation (opposite to current) that cancels some part of the driving electrokinetic field from the solenoid. From this it is seen that the torque on the charged cylinder is no longer created just by E_{k1} but now by $E_{k1} - E_{k2}$. Since the rotating charge represents a current a magnetic vector potential A2 is also created that cancels a portion of A_1 used to calculate the electrokinetic field at the cylinder so that this calculation is thus also reduced.

Of course our interest here is in the distribution of energy and momentum both appearing as kinetic mechanical values related to the motion and mass of the cylinder but also as field energy and momentum stored in space as a result of operation of the apparatus. We can note for example that angular momentum begins with a value zero and must maintain that value throughout operations. Therefore the angular momentum stored in the spinning cylinder must at all times be exactly canceled by an opposite angular momentum somehow stored in the fields of space to conserve angular momentum to a null value.

To examine these effects more closely we begin with a calculation that relates the motion of the cylinder to it's apparent current. An electric current is defined as the net rate at which charge passes any point.

$$I = \frac{dq}{dt}$$

And in this case we observe that if we designate a reference mark relative to our rotating charged cylinder, for every rotation of the cylinder the total charge q of the cylinder passes the mark. We designate the angular velocity of the cylinder as ω where:

$$\omega = \frac{d\theta}{dt}$$

Since each revolution of the cylinder represents 2π degrees we find that the current represented by the spinning cylinder is given by:

$$I = q\frac{\omega}{2\pi}$$

Since the charge q is a constant any time rate of change of the current is the result of a time rate of change of the angular velocity of the cylinder. Since our charged cylinder when spinning more or less can approximate a long solenoid with a single turn over it's length l, where l/L represent the fractional unit length of the cylinder, we observe that it generates a magnetic field B_2 within its interior given by:

$$B_2 = \mu_o\left(\frac{l}{L}\right)q\frac{\omega}{2\pi}$$

An important feature is that this B field is opposed to that created by the driving solenoid and therefore reduces the inductive stored energy in the magnetic fields. the spinning charged cylinder not only represents a current that creates a magnetic field, that stores energy, but also represents mechanical stored energy as well as angular momentum.

An Interesting Calculation: Two Charged hoops

Jefimenko calculates and interesting related problem that we mentioned in discussing action-reaction that provides some insight into this situation.[77] This involves two charged coaxial rings in the same plane in space, one of mass m_a and radius a, and a larger one of mass m_b and radius b. For purposes of this calculation, he assumes that b >> a. [Fig 18.] Consideration of the Electrokinetic fields created by the increasing current represented by an accelerating hoop shows that as one hoop is rotated in one direction the other hoop rotates in the opposite direction.

Fig. 3.11 When one of the two charged rings is rotated, the other ring starts to rotate in the opposite direction.

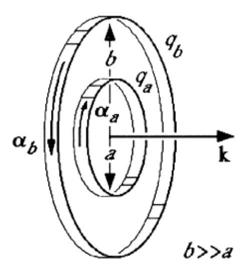

Figure 21 Jefimenko 3.11

[77] Jefimenko, Oleg, D. , " Causality, Electromagnetic Induction and Gravitation", Electret Scientific Co. Star city, 2000, pp 56-57.

Note that there is a temptation here to assume that somehow there is conservation of angular momentum between the two hoops resulting in equal amounts of angular momentum in each, but since the applied angular acceleration is mechanical, this is *not* the case. The angular acceleration of hoop a due to an acceleration of hoop b is not the same as the acceleration of hoop b due to an acceleration of hoop a except in certain special circumstances.

Jefimenko calculates the angular acceleration of hoop a due to an acceleration of hoop b as:

$$\vec{a}_a = -\mu_o \frac{q_a q_b}{8\pi\, m_a\, b} \vec{a}_b$$

While the angular acceleration of hoop b due to an acceleration of hoop a is given by:

$$\vec{a}_b = -\mu_o \frac{q_a q_b a^2}{8\pi\, m_b b^3} \vec{a}_a$$

Note that these are only equal when charges, masses and radii are exactly equal. Such a case seems to represent a kind of electro-mechanical "non-inductive" situation where all magnetic fields cancel and hence neither energy nor momentum is stored in space. And furthermore since angular accelerations are equal and opposite, like out motor-flywheel we discussed before, these non-inductive hoops store no magnetic energy nor total angular momentum, but can and do store mechanical kinetic energy.

From this we see in this situation the storage of energy and angular momentum in space relates to the *difference* between the magnetic fields created by the two spinning charged hoops. When we drive that difference to zero, then the total magnetic field tends to zero, which implies no stored electromagnetic energy or angular momentum although clearly the rotating masses store kinetic energy and mechanical angular momentum.

From the foregoing discussion it can be seen that the mathematical tools are available for a calculation of the actual velocity of the cylinder in our modified Feynman apparatus even taking into account the effects of the current represented by the rotating charge. We will not examine it in that detail here, but it should be clear that with some effort it could be done.

A Paradox in the Paradox: A Cylinder *Outside* the coil

Given the requirement of the Poynting vector that *both* an electric and magnetic fields are required for energy flow, we are going to slightly alter our apparatus. The original configuration has the charged cylinder just inside the bore of the solenoid. Now slightly change the apparatus placing the cylinder just *outside* the solenoid. If the cylinder is close to the solenoid in both cases, this does not change the above electrokinetic field considerations as the fields are identical just inside and just outside the solenoid. It is obvious from experience with transformers and similar devices that this modification will work little different from the previous inside configuration. The cylinder will acquire energy and angular momentum from a rising or falling current in the solenoid. The electrokinetic E_k field providing torque on the charged cylinder is virtually identical just inside and just outside the driving solenoid coil.

But the slight modification has caused a Poynting disaster. Now if the coil is of high conductivity and grounded it acts as a Faraday shield and there is no charge and hence electrostatic field inside the coil where the magnetic field is located. There is only an electrokinetic electric field from the changing current in the coil and we have already noted that in this case the energy flow is into and out of the space inside the coil and is radial and does not circulate. Also, there is no magnetic field outside the solenoid except for leakage. And one can argue that exchanging the long solenoid for a toroid can minimize leakage. Hence, since angular momentum stored in space would seem to require *both* and electric and magnetic field producing Feynman's circulating Poynting vector in the steady state of a flowing current, there can be found no circulating energy that one can assign to momentum in a field. Yet the cylinder will still experience torque when the current increases from zero or falls to zero from a fixed value just as before and the rotation as current ceases as in the Feynman case shows that *somehow* there must be angular momentum stored in space. This is a conundrum.

But Feynman himself hints at a suggestion in his discussion of the Aharonov-Bohm experiment:[78]

"*You may remember that for a long solenoid carrying an electric current there is a B-field inside but none outside, while there is lots of A [magnetic vector potential field] circulating around outside as shown in Fig. 15-6.*" [Fig 19.]

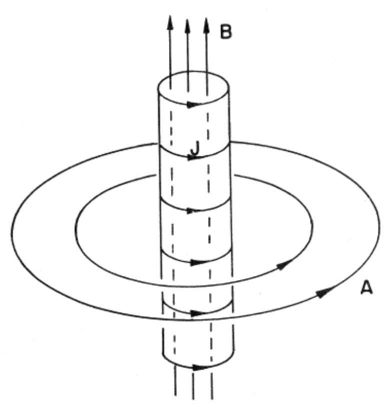

Fig. 15–6. The magnetic field and vector potential of a long solenoid.

Figure 22. Feynman 15-6.

[78] Feynman op. Cit., Section 15-5 p. 15-11

So not only have we missed a field outside the solenoid, but it also represents some kind of circulation as well. Even more interesting is that the A field is in the "right" direction. By that we mean that if we put an increasing current into the solenoid so the cylinder begins to rotate against the current, the A field is indicating rotation with the current such that momentum stored in space could cancel the mechanical angular momentum gained by the cylinder. Thus, there is the possibility that the total angular momentum remains conserved and zero as it was before we started. Similarly if the current falls, the A field collapses and the cylinder stops, losing all it's mechanical angular momentum while the collapsing A field represents the loss of all angular momentum stored in space which at any point in time leaves the zero angular momentum with which we started. So we may have found a field at least *indicating* angular momentum stored in space, particularly since we have already calculated above a connection between a change in the A field and a change in angular momentum and we have already noted that the integral of A around a closed path gives the magnetic flux through that loop.

Feynman notes that historically,

"*But because in classical mechanics A did not appear to have any direct importance and furthermore, because it could be changed by adding a gradient, people repeatedly said that the vector potential had no direct physical significance – that only the magnetic and electric fields are "right" even in quantum mechanics.*"

But not only the Aharonov-Bohm experiment but Jefimenko has also suggested that an A field in spite of its minimal interactions with charge is a real, measurable field rather than some mathematical trick.

Beginning with the equation relating A to the Electrokinetic field:

$$[\vec{\boldsymbol{A}}] = -\int \vec{\boldsymbol{E}}_{\boldsymbol{k}}\, dt \; + \; const\,.$$

where [A] is the retarded vector potential and E_k is the induced electrokinetic electric field. He continues:

"*Let us call the time integral of E_k the <u>electrokinetic impulse</u>. We can say then that the magnetic vector potential created by current at a point in space is equal to the negative of the electrokinetic impulse produced by this current at that point when the current is switched on. Since the electrokinetic impulse is, in principle, a measurable quantity, we thus have an operational definition and a physical interpretation of the magnetic vector potential.*"[79]

[79] Jefimenko, Oleg D., "Causality Electromagnetic Induction and Gravitation" ` 2nd Edition. Electret Scientific Co. Star City, WV, 2000, sec 2-4, p. 31. Jefimenko also suggests a related interpretation of the magnetic vector potential given by Emil J. Konopinsky, "Electromagnetic Fields and relativistic Particles" McGraw-Hill, New York, 1981, pp. 158-160.

In this way it is theoretically possible to build an "**A**" meter, which can measure the existence of the vector potential as a "real" field. In a manner similar to the way in which Faraday's law produces correct answers to many (but not all) problems even though **E** and **H** or **B** and **A** fields do not cause each other, there is a redundancy link between a magnetic field **B** and the vector potential **A** given by:

$$B = \nabla \times A$$

plus, we have also shown the relationship:

$$E_k = -\frac{\partial [A]}{\partial t}$$

Where [**A**] is the retarded vector potential, but these correlations do not represent causal links because all quantities are simultaneous. Indeed, the same electric current creates all three quantities, **A, B,** and $\mathbf{E_k}$, simultaneously and it is obvious that all three fields are therefore retarded from their single current source according to the distance to the observation point and the speed of light in vacuum.

To show that it is actually possible to measure **A** outside a long solenoid bearing a constant current where **E** and **B**, but not **A** are zero we note our previous calculation that showed outside such a solenoid:

$$A_\varphi = \left(\frac{N}{L}\right)\frac{\mu_o}{2}\frac{R^2}{r_o} I = \frac{K}{r}$$

Where we simply group the unchanging terms into a constant "K" and our observation point is at r, which was previously the diameter r_o of our charged cylinder. Since the magnetic vector potential only varies with distance, r, we want to give our observation point a velocity v_r or we replace r with $(r - v_r t)$ or:

$$A_\phi = \left(\frac{N}{L}\right)\frac{\mu_o}{2}\frac{R^2}{r_o} I = \frac{K}{(r - v_r t)} \text{ and } T = (r - v_r t)$$

By our above relationship between the electrokinetic field and the negative time rate of change of the magnetic vector potential we need to calculate:

$$\boldsymbol{E_k} = -\frac{\partial A}{\partial t} = -\frac{\partial A}{\partial T}\frac{\partial T}{\partial t}$$

Where

$$\frac{\partial A}{\partial T} = \frac{\partial K T^{-1}}{\partial T} = \frac{-K}{T^2}$$

And

$$\frac{\partial T}{\partial t} = -v_r$$

So that

$$\frac{\partial A(r,t)}{\partial t} = \frac{v_r K}{T^2}$$

Thus

$$E_k = -\frac{v_r K}{(r - v_k t)^2} = -\left(\frac{N}{L}\right)\frac{\mu_o}{2}\frac{R^2 I v_r}{(r - v_r t)^2}$$

Therefore it is clearly observed that motion in a non-uniform A field (but one which has no curl such that **B** is zero) indeed produces an electrokinetic $\mathbf{E_K}$ field capable of physical detection. This demonstrates the practicality of construction an "**A**" meter capable of measuring static **A** fields unlike Jefimenko's "impulse" methods. This is essentially the magnetic vector potential equivalent of a generating voltmeter used to measure static **E** fields. Therefore it is possible to measure static **A** fields outside of long solenoids or toroids. This calculation also suggests that the deflection of electrons moving past a long solenoid where there is no **B** field, but only an **A** field to deflect them in the **Aharonov–Bohm** experiment may actually be a classical rather than quantum-mechanical effect.

Note that in the case of a current-carrying toroid free of external **E** and **B** fields the above considerations relate to what is termed Anapole moments.[80] However, we will not discuss these moments or the issue of how **A** could be an indicator of momentum stored in space further at this time.

So a critical observation Feynman (and we) has made is that while **B** is zero (outside the solenoid), **A** has a non-zero value. Hence **A** exists even when **B** is zero. And interestingly, a change in **A** is related to an **E** field that is the electrokinetic EK field but the curl of **A** is also related to the magnetic field. Hence the key question would be if a circulating magnetic vector potential somehow represents angular momentum stored in space?

We know that

$$\vec{\boldsymbol{\tau}} = \frac{d\vec{\boldsymbol{L}}}{dt} = \left(\frac{d\vec{\boldsymbol{r}}}{dt} \times \vec{\boldsymbol{p}}\right) + \left(\vec{\boldsymbol{r}} \times \frac{d\vec{\boldsymbol{p}}}{dt}\right) = \vec{\boldsymbol{r}} \times \frac{d(m\vec{\mathbf{v}})}{dt} = \frac{d\vec{\boldsymbol{L}}}{dt}$$

Which is analogous to the basic relation of force being equal to the time rate of change of momentum. Comparing to linear momentum we know that:

$$Force = q\boldsymbol{E_k} = -q\frac{\partial \boldsymbol{A}}{\partial t} = \frac{dp}{dt}$$

From this equation we note a somewhat analogous role between linear momentum and the magnetic vector potential. But here the value of **A** at the charge relates to the total momentum imparted to the charged object. Unlike the Poynting vector, **S**, it is not a

80 See Wikipedia: http://en.wikipedia.org/wiki/Anapole_moment#History

representation of a momentum density in space which is to say a representation of storage in space. We do know that the energy that will drive the hoop outside the coil when the current falls to zero is stored in the interior of the coil. Hence it seems reasonable that the angular momentum that will be imparted to the hoop when the energy falls to zero comes from that same location and not from fields exterior to the coil. The only logical conclusion is that while the circulating **A** field certainly indicates stored angular momentum somewhere, that place would appear to not be outside the coil but rather within it.

But this leaves the question of just how that momentum and energy are transmitted from the interior to the exterior of the coil where the absence of a **B** field precludes the existence of a Poynting vector to provide such transport! Thus, it would appear that the relationship of momentum to charge times the magnetic vector potential outside a coil at a point is only another electromagnetic redundancy in the same manner as the induced emf is related to the magnetic flux through the circuit even though no magnetic field need exist at the wire where the voltage is induced. The exact mechanism and model of how exactly a changing current induces an electrokinetic electric field in space about itself remains a mystery. And the paradox remains unexplained.

So we might consider a variation of our experiment to examine linear rather than angular momentum. This is more straightforward because we already observed that the Poynting vector represents a linear momentum density in space and that vector also represents power flow in space. Therefore we propose a different experiment whereby we eliminate our rotating hoop either inside or outside the coil and instead place two charged objects on either side of the coil at some point. One object has a positive charge q while the other has a negative charge -q. These might be the plates of a capacitor placed on either side of the coil.

If we examine the force on this charge distribution (we assume the two charges are somehow fastened together and aren't allowed to move together by mutual attraction etc.and are thus in the same inertial frame) we find that any change of current in the solenoid creates electrokinetic fields outside that produce forces on both charges in the same direction away from the coil. Hence a changing current will cause the capacitor to be subjected to linear momentum and fly off into space.

But since we have stated that the magnetic field outside a long solenoid or toroid is zero, it is clear that the Poynting vector in this region is also zero meaning that there can be no energy or momentum imparted to the charged capacitor. Clearly there is a problem.

Hence, like Sherlock Holmes, we must deduce that somewhere our logic has failed since the forces which can’t exist clearly DO exist. The only possible explanation is that for changing currents the magnetic field outside long solenoids or toroids is not zero! If we ask how it is that the magnetic fields outside a toroid or long solenoid come to be zero we find that this comes about from a Biot-Savart calculation upon the coil. Each current element is producing a magnetic field and when all such current elements are summed over the entire coil it is found that inside the coil the magnetic fields add together producing a strong field while outside the coil they cancel producing an apparent zero magnetic field.

But something (as usual) has been ignored! And that something is retardation. When you apply a changing current to a wire that wire is in effect a transmission line and the disturbance can travel down that wire no faster than the speed of light. Furthermore, if you pick an arbitrary observation point outside the coil the distances from that point to the various current elements of the coil are a variable distance. So once again your current changing disturbances traveling at finite speed to the observation point arriving at different times. In short we argue that while there are changing currents, the magnetic field and hence the Poynting vector is not zero. We can surmise that the values that occur correspond to the energy and momentum imparted to our charged objects.

This is not something new. Afanasiev[81] has calculated energy from a toroidal solenoid driven by a time-changing current. He notes:

We conclude: time dependence of the solenoid current generally leads to a non-vanishing magnetic field outside the solenoid. The flow of electromagnetic energy is directed off the solenoid.

Which indicates a Poynting vector in the region outside the solenoid as a candidate for the transfer of energy and momentum to our charged objects. He concludes:

In fact the remarkable properties of the toroidal solenoid are practically unknown and the matter presented here and in [another paper by Afanasiev from 1986] fill this lacuna. From the non-vanishing of the magnetic field outside the solenoid it follows that the transition process to the stationary regime in the Aharonov-Bohm effect theory should be reconsidered.

Note that his last suggestion to reconsider the Aharonov-Bohm experiment is similar to our own suggestion above and precedes ours by more than two decades. However, an examination of the complex details of the time-dependent fields external to toroidal solenoids is well beyond the scope of this discussion.

Force or Change of Momentum?

A question that interested Jefimenko very much was the role that forces played in defining fields. He wondered if a better understanding could be obtained through an examination of the role of momentum transfers instead, since as we noted above:

$$\vec{\boldsymbol{F}} = \frac{d\,\vec{\boldsymbol{p}}}{dt}$$

and EM fields clearly represent storage and transfers of momentum in space.

81 Afanasiev, G. N., J. Phys. Math. Gen. 23 (1990) 5755-5764.

Jefimenko in Appendix 3 of his book[82] examines these questions. He applies the redundancy in electromagnetics to calculate the so-called “forces” on both charges and currents. For example, he supposes two oppositely charged dielectric plates and tries to find the force between them. He does this using the Lorentz equation

$$\vec{F}=\rho\int(\vec{E}+\vec{v}\times\vec{B})\,dV$$

Which assumes fields and forces, but then also calculates the *same* result using the electric scalar potential, the vector electric potential and the Maxwell stress integral. However, in spite of identical numerical results he notes that conceptually the “force” is acting at different locations in each case. Jefimenko then computes the force between two current-carrying wires using the Lorentz equation, the magnetic scalar potential, the magnetic vector potential and the Maxwell stress integral, Again he obtains identical numerical results in each case, but philosophically the force is seen to be acting in different locations depending on the method. This is the redundancy in electromagnetics. This is how Faraday's law can be used to calculate induced emf from a changing magnetic field even though there is no magnetic field at the point where emf is calculated. Therefore the question arises as to which calculation represents the “real” force and which are simply redundant calculations giving a correct answer. So Jefimenko asks the question just *how* is energy transferred from electromagnetic fields where it is stored in space into the kinetic energy of charges placed in motion?

He does this by noting that a flow of energy is related to the Poynting vector by the equation:

$$\frac{dU}{dt}=\oint\vec{S}\cdot\vec{dS}$$

where **S** is they Poynting vector, **S = E x H** and d**S** is the element of the surface enclosing the charge distribution that is receiving the energy. Since the energy flow above was taken as *into* the charge distribution, Jefimenko switches to the standard flow *out* of the distribution that gives:

$$\frac{dU}{dt}=-\oint\vec{S}\cdot\vec{dS}=\oint\vec{E}\times\vec{H}\cdot\vec{dS}=-\oint\vec{E}\cdot\vec{H}\times\vec{dS}$$

Using Gauss' theorem to change to a volume integral he obtains:

$$\frac{dU}{dt}=\vec{E}\cdot\int\nabla\times\vec{H}\,dV$$

Then using Maxwell's equation

$$\frac{dU}{dt}=\vec{E}\cdot\int\rho\vec{v}\,dV$$

82 Op. Cit, Jefimenko, Oleg D., “Electromagnetic Retardation and the Theory of Relativity”, Appendix 3.

Where ρ is the charge density of the distribution and **v** is it's velocity. The final result is obtained:

$$\frac{dU}{dt} = q\vec{\boldsymbol{E}} \cdot \vec{\boldsymbol{v}}$$

Jefimenko then observes that this result was obtained with no references to “force” whatsoever. He then concludes: “*According to our calculations, the kinetic energy that the charge q receives from the electric field in which it is located does not involve any force action at all and occurs entirely due to an energy influx into q via the Poynting vector*.”

Is this a reasonable conclusion or has Jefimenko himself fallen victim to electromagnetic redundancy? Time and further investigations will hopefully tell, but this does point out the danger of assuming pat assumptions of causality such as a changing magnetic flux “causing” an emf without a very careful examination of what actually causes what.

For this reason Jefimenko looks at linear momentum transfers as a source of electromagnetic forces. We have already examined the Maxwell stress equation connecting the rate of change of mechanical linear momentum and EM fields:

$$F = -\varepsilon_o \mu_o \frac{\partial}{\partial t} \int S \; dv - \left[\frac{\varepsilon_o}{2} \oint E^2 \; dS - \varepsilon_o \oint E \left(\vec{\boldsymbol{E}} \cdot \vec{\mathbf{dS}} \right) \right] - \left[\frac{\mu_o}{2} \oint H^2 \; dS - \mu_o \oint H \left(\vec{\boldsymbol{H}} \cdot \vec{\mathbf{dS}} \right) \right]$$

Where we can express the force as a change of momentum:

$$\frac{d\,\vec{\boldsymbol{p}}_{MECHANICAL}}{dt} = \vec{\boldsymbol{F}}$$

In this equation the **E** and **H** fields are the *total* fields in space which include any external fields as well as the fields created by the charges and currents themselves. And while the above equation is usually presented as showing conservation of momentum derived from the Lorentz force equation, Jefimenko asserts that it is a fundamental equation in it’s own right, linking electromagnetic and mechanical momentum. Indeed, he shows that the Lorentz force equation can be derived from it:

We apply the following general vector identity to the above surface integrals:

$$\frac{1}{2} \oint A^2 dS - \oint A \left(\vec{\mathbf{A}} \cdot d\,\vec{\mathbf{S}} \right) = \int \left[\vec{\mathbf{A}} \times \left(\nabla \times \vec{\mathbf{A}} \right) - A \left(\nabla \cdot \vec{\mathbf{A}} \right) \right] dV$$

to obtain:

$$\frac{d\,\vec{\boldsymbol{p}}_{\boldsymbol{M}}}{dt} = -\frac{1}{c^2}\int\frac{\partial}{\partial t}\left(\vec{\boldsymbol{E}}\times\vec{\boldsymbol{H}}\right)dV$$
$$+\int\left[\varepsilon\left(\nabla\cdot\vec{\boldsymbol{E}}\right)\vec{\boldsymbol{E}} + \mu_o\left(\nabla\cdot\vec{\boldsymbol{H}}\right)\vec{\boldsymbol{H}} - \varepsilon_o\vec{\boldsymbol{E}}\times\left(\nabla\times\vec{\boldsymbol{E}}\right) - \mu_o\vec{\boldsymbol{H}}\times\left(\nabla\times\vec{\boldsymbol{H}}\right)\right]dV$$

And then from Maxwell's equations we substitute:

$$\varepsilon_o\nabla\cdot\vec{\boldsymbol{E}} = \rho,\;\; \mu_o\nabla\cdot\vec{\boldsymbol{H}} = 0.\;\; \nabla\times\vec{\boldsymbol{E}} = -\frac{\partial\mu_o\vec{\boldsymbol{H}}}{\partial t},\;\; \nabla\times\vec{\boldsymbol{H}} = J + \frac{\partial\varepsilon_o\vec{\boldsymbol{E}}}{\partial t}$$

to obtain:

$$\frac{d\,\vec{\boldsymbol{p}}_{\boldsymbol{M}}}{dt} = -\frac{1}{c^2}\int\frac{\partial}{\partial t}\left(\vec{\boldsymbol{E}}\times\vec{\boldsymbol{H}}\right)dV$$
$$+\int\left[\rho\vec{\boldsymbol{E}} + \varepsilon_o\vec{\boldsymbol{E}}\times\left(\frac{\partial\mu_o\vec{\boldsymbol{H}}}{dt}\right) - \mu_o\vec{\boldsymbol{H}}\times\left(\vec{\boldsymbol{J}} + \frac{\partial\varepsilon_o\vec{\boldsymbol{E}}}{\partial t}\right)\right]dV$$

But since:

$$\varepsilon_o\mu_o = \frac{1}{c^2}\;\; and\;\; \vec{\boldsymbol{H}}\times\frac{\partial\varepsilon_o\vec{\boldsymbol{E}}}{\partial t} = -\frac{\partial\varepsilon_o\vec{\boldsymbol{E}}}{\partial t}\times\vec{\boldsymbol{H}}$$

the time derivative terms cancel leaving:

$$\frac{d\,\vec{\boldsymbol{p}}_{\boldsymbol{M}}}{dt} = \int\left(\rho\vec{\boldsymbol{E}} - \mu_o\vec{\boldsymbol{H}}\times\vec{\boldsymbol{J}}\right)dV = \int\left(\rho\vec{\boldsymbol{E}} + \vec{\boldsymbol{J}}\times\vec{\boldsymbol{B}}\right)dV$$

which is the Lorentz force equation expressed in terms of a rate of change of mechanical momentum of the charge and current distribution subjected to fields **E** and **B**. This implies that our original stress equation is a fundamental relation between mechanical momentum and electromagnetic fields. But how can changing momentum create mechanical forces, asks Jefimenko? He speculates that the action is much like the way a ballistic pendulum converts impacts to force, but does not speculate on EM fields being some kind of projectiles. He concludes:[83]

"*The examples and calculations presented in this Appendix show that force in electric and magnetic systems is a convenient and important mathematical device, but not the physical effect, entity, or agent as we know force in mechanics. They also show that in electric or magnetic systems there occurs a direct exchange of momentum between the electromagnetic field and charges or currents located in this field; this momentum exchange is perceived as an electric or magnetic force. Thus, what we call "force" in electric and magnetic systems is actually a surrogate for the momentum transfer phenomenon*."

[83] Ibid p. 321.

While Jefimenko concludes above that force is a mathematical convenience but not a real physical effect, we must again point out the very issues that Jefimenko himself raised with regard to electromagnetic redundancy and the interlocking nature of Maxwell's relations which require a careful determination of causality before any such assertions are made.

A Review of Causal Action in Electromagnetics

The Causal Equations for E and H

$$\vec{E} = \frac{1}{4\pi\varepsilon_o}\int\left(\frac{[\rho]}{r^2} + \frac{1}{rc}\frac{\partial[\rho]}{\partial t}\right)\vec{r}_u\, dv' \; - \; \frac{1}{4\pi\varepsilon_o c^2}\int \frac{1}{r}\left[\frac{\partial \vec{J}}{\partial t}\right] dv'$$

and

$$\vec{H} = \frac{1}{4\pi}\int\left(\frac{[\vec{J}]}{r^2} + \frac{1}{rc}\frac{\partial[\vec{J}]}{\partial t}\right) \; x \; \vec{r}_u \, dv'$$

To review, the basic unit of action in electromagnetics is charge. Charge comes in two types with opposite forces. An unmoving static charge creates forces on other charges that are described by force field mathematics giving radial forces between the two charges and a spherically radial distribution of the force field about a given "point" charge. This action is not instantaneous but travels at the speed of light or slower through space. This is called a "retarded" field. Maxwell likened this action to a stress in space. And he also suggested that if space were like a set of bedsprings, which could become "stressed" by the electric force fields, it could store energy, momentum and even with the right geometry angular momentum. He calculated this "stress".

Although charge is conserved and can only be created or "destroyed" in positive-negative pairs so the net charge in the universe is always zero, if a region of space finds the amount of charge changing with time, the static field is modified by an amount related to the time rate of change of the charge. Increasing charges add more field (or one can look at it as a second radial field because it varies differently with distance) and decreasing charges add a negative field, which is to say reduces the original radial field. The original field is related to the amount of charge present while the modifying field is given by the time derivative of the amount of charge. Both fields are radial from the charge and retarded. This is an "ordinary" or what we have termed an electrostatic field even if it is changing with time. Looking at the Jefimenko causal Maxwell equations we find that they suggest that electrostatic phenomena is not particularly mathematically entwined with the other terms. This perhaps suggests two separate phenomena at play.

And there is another important phenomena in electromagnetics as well. Charge can flow from one point to another creating a current. This current can be created with, say, electrons down a wire or even an electron beam in space. But an interesting fact is that a current can be created with a situation where negative charges are moving one way and positive charges moving the other. Both motions represent an identical current because the direction of motion and sign of the charged carriers both change together. And the interesting thing is that one can then integrate charge over a volume of space containing the current and find a zero charge because the positive and negative charges cancel. And because there is no average charge, the electrostatic force fields we just discussed will all be zero. But in spite of there being zero total charge the currents still can produce motion of other charges as well as of other currents. And furthermore since charge-neutral currents are producing force fields in space, these fields *also* represent stresses in space and hence, stored energy, momentum and angular momentum.

A steady current in space creates a force field about itself that is quite different from an electrostatic force field. This force field is also retarded and travels at the speed of light or slower. This force field is termed a magnetic field and is described by mathematics and actions quite different from the electrostatic fields. For example while an electrostatic field is radial and is termed an *irrotational* field, the magnetic field is found to exist in closed loops about the current elements and is termed *solenoidal*. A charge moving at a constant velocity at right angles to a magnetic field experiences a sideways force at right angles to *both* velocity and the field. This is termed the Lorentz force relation.

Looking at our causal Maxwell equations we see that for a steady element of current there is created about it a magnetic field in circles. This field is not uniform over all angles as in the electrostatic case, but falls off as the sine of the angle between the axis of the current direction and the observation point. Hence, there is no magnetic field produced in either direction along the axis of the current motion. Such a magnetostatic field is termed the Biot-Savart field for historical reasons. This magnetic field falls off as the inverse square of distance as does the electrostatic field above. Further more it has been observed that there are forces between currents that can attract and repel currents. This force field is also described by the magnetic field and is retarded. Forces between currents can be “explained” by the Lorentz force equation. Parallel wires with currents in the same direction attract each other and those with currents in opposite directions repel.

In our causal equations we also observe that in a manner completely analogous to that found with charge, the Biot-Savart static magnetic field is modified or has a second magnetic field added or subtracted as a result of the time rate of change of current. Furthermore this added field from the time rate of change falls off with distance (also the case in electrostatic fields) at a different rate from the static field. The static field falls off rapidly as $1/r^2$ while the time rate of change field falls off more slowly as $1/r$. The result is that at great distances only the derivative fields remain. We have termed this derivative field the *magnetokinetic* field since only a current changing in time creates it. This field like the Biot-Savart field involves the same cross product between the current direction and a unit vector in the direction of the observer, which means that the magneto-kinetic field is also zero along the axis of the current.

Looking at the causal equations we observe that currents also can create an electric field. And that field is also produced by the time rate of change of current. It has been therefore termed the *electrokinetic* field. This vector field is spherically symmetric about any current element changing with time. And the direction of the electric field vector is parallel or anti-parallel to the axis of the current element. There is also a negative sign related to Lenz's law that says that the electrokinetic field produced by an increasing current is in a direction such that it *opposes* the electric field causing the current to flow. On the other hand if the current is decreasing the electrokinetic field produced by that current on itself will be in the direction of the current-driving field and will tend to keep the current flowing. Generally speaking these effects are termed self-inductance.

In the past, calculation of self-inductance has been a problem because while we can easily calculate the electrokinetic field produced by the current in one current element reducing the driving field in other nearby elements, we notice that the electrokinetic field of a current element on itself seems to become undefined due to the distance term in the denominator going to zero. But this is not what actually happens. In truth, it is a massive feedback effect with the electrokinetic fields of each current element producing current reductions in all the other current elements including its own. The electrokinetic field of a current element upon itself does not produce an infinite electrokinetic field, but rather simply tries to drive the current in that element to zero. This would appear to say that no currents could flow and inductance would be infinite, but there are also other elements off to the side that are not connected by unity in operation. It's all a massive interconnected matrix. When the driving field is driven to zero by an induced electrokinetic field, the current also drops to zero, which of course reduces the electrokinetic field to zero! Hence the cybernetic feedback nature of this effect is clearly seen. However, we know that for straight infinitely thin wires (where there is no contribution from elements off to the side) the inductance is indeed very high (so-called "infinite") which implies that it takes large voltages (high electric fields) to force a current through such a wire. This is obviously how inductance becomes only a function of conductor geometry.

In the discussions above we pointed out the redundancy of values in electromagnetics. For example, one can calculate the electrokinetic field by use of Faraday's Law, which relates the magnetic flux created by a current to the potential created in space by the electrokinetic field. However, the causal equations show that an electrokinetic **E** field is the *direct* result of a changing current. No magnetic fields whatsoever are involved in the process. The changing current *somehow* creates an electric field about it itself and *also* simultaneously creates a magnetic field around itself in some manner. Both these effects are intimately related and both are retarded from the changing current action. Because of this interrelationship, Faraday's Law gives correct answers, but clearly it is not a changing magnetic flux (field) that is causing the electrokinetic field, because that field can exist quite nicely outside of a toroid where there is no magnetic field whatsoever.

But changing currents also create another retarded field: The Magnetic Vector Potential. It was shown above that electrokinetic field is given by the negative time derivative of the magnetic vector potential. Both vectors are retarded equally from their source currents, which is clear because in a derivative equation (like some of Maxwell's) both sides are simultaneous in time.

So at this point we observe that currents produce a whole host of interrelated fields and electromagnetic phenomena. A current creates a Biot-Savart magnetic field retarded from the current and in a vacuum travels at the speed of light. This field is proportional to the current and falls off with the inverse square of distance. A changing current also creates a second magnetokinetic field that adds or subtracts from the former static field, which is proportional to the time rate of change of the current. This magnetic field falls off with the inverse of distance. The same current also creates another field. This is the retarded magnetic vector potential. This field is proportional to the current and is created spherically about a current element with a direction parallel to the current. This field falls of with inverse distance and can exist where the above magnetic fields are zero such as outside a toroid or long solenoid. And lastly that same current creates a retarded electrokinetic **E** field which we have shown above mathematically encompasses both electrokinetic induction $\mathbf{E_K}$ fields as well as Lorentz $\mathbf{E_L}$ effects.

The bottom line is that simply moving charge creates a whole host of interrelated electomagnetic relationships. But then we ask the question: If currents are simply moving charge then how does that charge get moving? The traditional answer is that an electric field of some type creates a force that moves the charge. But is this view a bit too pat for our careful examination of causality and interactions?

Consider for a minute the following situation:

An electrokinetic field is produced by some current and travels through space at the velocity of light to apply a force to some charged particle. That particle then moves under the influence of that force with some function of time f(x,y,z,t). We are used to thinking that the force also "causes" the velocity and acceleration of the particle, which are derivative relationships to the motion of it. In a sense that is true, but we observe that the motion of the particle is a real physical action. There is matter moving through space. But the velocity of the particle is simply a mathematical operation upon our function describing that actual motion. So is acceleration. Thus, we can see that velocity and acceleration are not real actions at all, but rather imaginary mathematical output and since this output is calculated from f(x,y,z,t) they are simultaneous with it. Hence velocity is actually *not "caused by"* motion but rather only calculated from it. Indeed if one looks at the mathematical operations to calculate velocity it is seen that it is based on taking limits that require information from the past, the future or both. Hence the causality of velocity is in question not only because it occurs simultaneously with a point in the motion function f(x,y,z,t) but also because it can be found with data from the future which violates the principle of causality.

A similar situation exists with the magnetic vector potential where the causal action is between a changing current and an electrokinetic field in space that is produced as a result, but there is a derivative relationship between the Electrokinetic field and the magnetic vector potential which are simultaneous in time. Therefore one is forced to ask if the magnetic vector potential is a "real" quantity or simply a mathematical imaginary one. For a long time now the magnetic vector potential has been regarded as simply a mathematical quantity useful as a "trick" for obtaining answers. But we discussed above Feynman's questioning of this long-held belief where he suggested that perhaps there is some evidence that the magnetic vector potential is the "real" field and perhaps a **B** field and the electrokinetic field are the imaginary mathematical calculations. It's an important question.

The fact that there are no “models” upon which we can hang our thinking caps to explain how a changing current produces an **E** field about itself in the same direction as the current or how a magnetic vector potential is also created about the current in the same fashion, together with the extensive redundancy of electromagnetic relationships does not make the job of sorting out what is actually causing what far from easy. However, these new ways of looking at the true causality of electromagnetic actions as suggested by Jefimenko, Feynman and others seem to hold great promise for a better understanding of the ancient electromagnetic phenomena.

We shall leave these questions at this point and note that the late professors Oleg Jefimenko and Richard Feynman have raised some very interesting new points about electromagnetics, which many have long been considered established and over and done with. Clearly there seems to be a wealth of concepts to be mined there especially given the advent of modern digital computation. Relations such as retardation known since nearly the beginning, but ignored because of calculation difficulties are now opened up to a good second look. While the simplified calculations have been productive by supplying much to 20th century civilization, the way pointed here is that there may indeed be much more left to dig out of Maxwell’s magnificent theory if one simply digs a bit deeper. Perhaps that will be a direction in the 21st century.

A topic that very much interested Jefimenko and Feynman was the nature of gravity. Jefimenko, expanding on the speculation of a co-gravitational field by Heaviside, develops an extensive mathematics that demands closer examination. In addition, Jefimenko develops a number of relativistic relationships starting only with the principle of relativity and electromagnetics. As he puts it:

“We shall develop relativistic electromagnetism solely on the basis of an electromagnetic retardation combined with the principle of relativity without any additional postulates, hypotheses, or conjectures.”

While neither relativity nor gravitation has been addressed here both topics appear to represent treasure troves of new ideas in Jefimenko’s works and anyone interested in those topics would do well to review his work and evaluate his speculations.

Note that at present, the late professor Jefimenko’s self-published books are readily available from Amazon.com and the author is indebted to Bill Miller for inducing him to purchase Oleg Jefimenko’s interesting books.

Note also that the author wishes to thank Neil Bates and Jos Bergervoet for the lively USENET discussions in sci.physics.electromag on these matters that very much helped to clarify a number of issues and especially Professor Kirk McDonald of Princeton who was kind enough to grant the author access to his massive database of classic

electromagnetic papers and who has graciously placed his own electromagnetic papers on the INTERNET for educational and research purposes.

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