Faraday's Law of Induction is Bogus

Michael Faraday (1791-1867) stands as one of the giants of early electromagnetic study. From a self-educated bookbinder apprentice he rose to become one of the most celebrated members of the Royal Society. But today more than a century later, two particularly erroneous items have been associated with Faraday. One would be his invention of the bogus "lines of force" idea that sprang from the suggestive patterns of iron filings when subjected to the action of magnets.[1] The idea persists to this day even though it is clearly not able to provide precise measurements in any reasonable fashion. The other idea bearing his name and creating great misunderstanding is "Faraday's Law of Electromagnetic Induction". Faraday was interested in "induction of various types and surmised that if an electrostatic charge could induce a secondary charge in a distant body, perhaps a current could induce a secondary current in a distant conductor. That guess proved to be true. And in fact, is a correctly stated description of the phenomena.

Fig 1. Michael Faraday

[1] Wittaker, Sir Edmund, F.R.S., " A History of the Theories of Aether and Electricity", Thomas Nelson and Sons Ltd., London, 1951, Ch VI, p170ff.

The problems, however began with the general acceptance of Faraday's law which can be written as:[2]

$$Emf = -\frac{d\Phi}{dt}$$

That relates the emf or voltage induced into a loop conductor to the summation of the magnetic field (which is termed "flux") passing through the center of that loop. It's a neat and very handy relationship that is commonly used in science and engineering. There's just one minor problem. It's not always correct. Sometimes it doesn't work at all and when it does it creates a very wrong impression.[3] That wrong idea is that somehow the changing magnetic flux through the loop is *causing* the induced voltages. That such an assumption is wrong can be demonstrated by noting that causality is a natural law on the earth. If two actions happen at the same time they cannot be causing each other! And that is especially true if any distance separates two actions since actions that can transmit information have never been observed to exceed the speed of light. We notice with regard to the above "law" that the magnetic field need not even be present at the location of the voltage induction, as happens with a loop of wire outside a toroid. Thus, a change of flux at the *center* of a wire loop purportedly causes induced emf out at the loop, outside the region where the flux is even present. This is nothing more than the old idea so common in Faraday's day termed "action at a distance". Since this idea defies causality, it has long been discredited as bogus. Nevertheless, it somehow seems to persist in physics renamed "nonlocal" interactions. But these days such arguments are usually reserved for quantum mechanics and are not given a second thought in Faraday's law.

Even Maxwell said:[4] "When the number of lines of magnetic induction which pass through the secondary circuit in the positive direction is altered, and electromotive force acts round the circuit which is *measured by* the rate of decrease of the magnetic induction through the circuit" [emphasis ours]. Note that he said the induced voltages were "measured by" not caused by the change in the magnetic field! But Faraday, himself, only proposed causality between a current and a secondary current at a distance. He never implied that a magnetic flux created the induced emf. So suppose we consider a short wire of length dL carrying a current I.

We actually do know what happens in that case. The first thing we observe is that there is set up in space about that current element what is termed a "magnetic induction". Today this is generally termed a magnetic field, although "field" is actually a mathematical term used to describe a vector quantity that may vary in magnitude and direction over a region of space. But

[2] Resnick, Robert, and David Halliday, "Physics", John Wiley and Sons Inc., New York, 1960, p. 741.

[3] Feynman, Leighton, Sands, "The Feynman Lectures on Physics", Addison-Wesley Co. Palo Alto, 1963, Section 17-2 Volume II.

[4] Maxwell, James Clerk, "Treatise on Electricity and Magnetis", Vol,II, Section 531, Dover.

to avoid confusion we will also term it a field and give it the standard vector symbol **B.** In a static case the law of Biot-Savart describes this magnetic field.[5] We know how it acts. It has a magnitude proportional to the current giving rise to it. It is formed in circles about the current in planes perpendicular to the direction of the current. It falls off in intensity as $1/R^2$ in distance from the current element and furthermore there is a variation in angle about the source current by the sin of the angle between the current and radius line to the observation point. This means that there is no magnetic field induced down the axis of a straight wire by the current in it. That implies that if a magnetic field is the cause of induction, that straight wires have no self-inductance. That statement is simply incorrect. Thus, this fact provides additional proof that Faraday induction has nothing to do with magnetic fields.

The way Faraday induction actually works is that a changing current source not only creates a magnetic field in the space around itself, but also creates an electric field. This is a real electric field capable of accelerating charged particles and the like. But it is known from causality that electric and magnetic fields cannot create each other.[6] It turns out the actual causal source is a current. A current element clearly sends forth *both* the magnetic field and the electric field of induction at the speed of light. Both travel in straight lines from that current source. Nothing follows the longer paths of the magnetic or electric "lines of force".

We also know how the electric field of induction works. It travels away from the current element in all directions at the speed of light. Just like the magnetic field from the same current source, it is therefore delayed when observed at a distance. This is termed being "retarded". The direction of the electric induction field vector is always the same as that of the source current. Always. And the **E** field exists in a sphere about the source current with no variation according to the angle the line to the observation point makes with the source current. And the magnitude of this electric field falls off as $1/R$ rather than the faster $1/R^2$ rate of a magnetic field. Finally, we note that the magnitude of this electric induction field is related not to the magnitude of the source current but rather to its time rate of change, which in calculus is termed it's derivative. For that reason, while the direction is always parallel to the current source if the current is decreasing, it will be anti-parallel that is to say face in the opposite direction to the current source if the current is increasing.

It is important to note here that this electric field induced by a changing current has quite different properties from a simple electrostatic field. For that reason we shall term the induced field $\mathbf{E_k}$ electrokinetic field that implies a source of "charges in motion" which is to say a current, while an ordinary electrostatic field $\mathbf{E_s}$ represents the usual Coulomb forces between stationary charges. The difference in properties can be seen for example in that the divergence of

[5] Resnick & Halliday, Op. Cit. p. 729.

[6] Jefimenko, Olg D., "Causality Electromagnetic Induction and Gravitation", 2nd Ed. Electret Scientific Co. Star City, 2000, p. 16.

$\mathbf{E_s}$ is equal to $4\pi\rho$ while the divergence of $\mathbf{E_k}$ is always equal to zero. Similarly the curl of $\mathbf{E_s}$ always equals zero while the curl of $\mathbf{E_k}$ = -dB/dt.

We also find there is a third retarded quantity traveling out from the current source. This is the magnetic vector potential, termed **A**, which can be thought of as a "stress" thrown into space at the speed of light. This "stress", like the $\mathbf{E_k}$ field of induction is spherical about the current source, does not depend on angle and likewise takes the direction of the current source. It also falls off with distance at the 1/R rate. However it depends on the value of the current rather than it's rate of change. The magnetic Vector Potential is defined as[7]

$$\vec{A}(x,y,z)=\frac{\mu_o}{4\pi}\int_v \frac{\vec{J}(x',y',z')}{R}dV'$$

That in the case of a thin wire becomes:

$$\vec{A}=\frac{\mu_o}{4\pi}\int\frac{\vec{I}}{R}d\vec{L}$$

where R is the distance from the observer to a given current element IdL. **A** is retarded so thus occurs in space at the same time as the retarded **B** and $\mathbf{E_k}$ fields. This means that **A** *cannot* be the "cause" of either **B** or $\mathbf{E_k}$. However the *value* of both those vectors can be calculated from **A**! We find that:

$$\vec{B}=\nabla\times\vec{A}$$

and

$$\vec{E}=-\frac{\partial\vec{A}}{\partial t}$$ [8]

Thus we see that while we can use **A** as a means to a solution of induction problems, it is clear that the actual electromagnetic source of both the magnetic field (induction), **B**, and the induced electric field, $\mathbf{E_k}$, is the current. **A** is not a source of either of these actions.

You might think at this point that these clarifications are not surprising given the more than century of physics that has elapsed since Faraday studied induction, but the true surprise is that these clarifications together with a true representation of induction was put forth by Franz Ernst Neumann (1798-1895), a contemporary of Faraday. While Faraday was attempting to explain induction using his own conception of "lines of force", Neumann started with Ampère's analysis.[9] Neumann also included in his analysis the law found by Emil Lentz, which gave an indication of the direction of induced currents. A major breakthrough of Neumann was the

[7] Plonsey and Colin, "Principles and Applications of Electromagnetic Fields", McGraw-Hill, 1961, p 204.
[8] Jefimenko, op. cit., p. 42.
[9] Whittaker, op. cit. p 198.

inclusion of the magnetic vector potential that it turns out was an analytic measure of what Faraday had termed the "*electrotonic state*". While we have already pointed out that the vector potential, **A**, is retarded and thus not a source of either **B** or $\mathbf{E_k}$, but nevertheless Neumann's formula can be divided in a way that shows that it stands correct.

Since the kind of electric induction we are discussing here involves only changing currents and no motion of apparatus, it can be called "transformer" induction. This is a separate issue from moving magnets and coils as found in electro-mechanical generators and the like. In this case if a current is impressed on the "primary" or "source" conductor of the induction device, a voltage will be found induced upon the ends of the open-circuit secondary or "target" conductor of the device. The relationship between these two quantities is given by the relationship:

$$V_{\sec odnary} = -M_{ij} \frac{dI_{primary}}{dt}$$

Where M_{ij} is termed the "mutual inductance" between the two circuits and is given by the Neumann formula:[10]

$$L_{ij} = \frac{\mu_o}{4\pi} \iint \frac{d\vec{L}_1 \bullet d\vec{L}_2}{R}$$

Because in "modern" texts this formula is often derived from the bogus Faraday's Flux law, it is often implied that the double integrals must be about two closed loops where one is the primary and the other is the secondary of the "transformer". But that is not necessary even when the non-causal vector potential, **A**, is incorporated into the formula. Therefore one calculates the induced secondary potential by first integrating the electrokinetic field along the secondary conductor giving:

$$V_{Secondary} = \int \vec{E}_k \bullet dL_S$$

At this point we note that although $\mathbf{E_k}$ is not caused by **A** in either it's retarded or ordinary form, it can be calculated from **A**. If the circuit parts are of smaller dimensions such that the delays produced by retardation can be neglected, ordinary vector potentials can be used, otherwise the retarded values of **A** should be employed. So substituting for $\mathbf{E_k}$ above we find:

$$V_{Secondary} = \int -\frac{d\vec{A}}{dt} \bullet d\vec{L}_S = \int \left[-\frac{d}{dt} \left[\frac{\mu_o}{4\pi} \int \frac{\vec{I}}{R} d\vec{L}_p \right] \right] \bullet d\vec{L}_S = \frac{dI}{dt} \frac{\mu_o}{4\pi} \iint \frac{d\vec{L}_P \bullet d\vec{L}_S}{R} = -L_{PS} \frac{dI}{dt}$$

[10] Plonsey & Colin, op. Cit. p. 275

Thus we have seen that the Neumann equation and not the Faraday relationship is the correct causal formula to calculate the induced voltages from any current distribution. The Vector potential drops out and is not needed. However the calculation of **A** is essentially the same mathematics as the calculation of $\mathbf{E_k}$ directly so the two techniques are the same. Furthermore it turns out that if the current is a ramp function, the retarded values are not needed no matter how large the dimensions.[11] The connection by Faraday of the change of magnetic flux through a circuit loop to the emf induced within that loop from anther current is really simply a lucky happenstance as both the magnetic field *and* the electrokinetic field are created by the same source current and thus are related but not causal of one another. This fact highlights more than a century of physics and engineering taking the “easy road” of assuming that if an equation states that the left side equals the right side, it somehow implies that one side *causes* the other quantity. Such assumptions are clearly unjustified without further proof.

Note that the Neumann equation not only easily explains the reciprocity law which makes $L_{sp} = L_{ps}$ but also explains the observation of Faraday that there is no induction between perpendicular wires

To elucidate these points I will now consider a simple induction situation that clearly demonstrates the difficulties with the Faraday flux solutions and the ease with which simple correct solutions can be obtained using the Neumann formula even in cases where the flux area is less than well defined.

We are going to build an induction “transformer”. It is to consist of a 1-meter length of *straight* coaxial cable. The cable consists of a superconducting foil outer conductor 4mm in diameter and a “very thin” superconductor center conductor. The dielectric separating these conductors is “vacuum” which in modern physics means it’s “nothing at all”. We are going to run a current down the OUTER conductor increasing at the rate of one ampere per second. The question then is what voltage, if any, appears between the two opposite ends of the center conductor (the circuit is open and only voltage is measured).

We will initially use the following assumptions to make the problem simple: We will assume that all connecting and measuring wires arrive at the coax at such right angles so as to not contribute to the induced voltage. All “end effects” can be neglected. The coax is assumed “long” which means its length is much longer than it’s diameter. No current is assumed to flow in the center conductor at any time beyond the electrostatic shifts needed to achieve a measurable potential. For that reason, by symmetry, and Ampere’s law we can see that the magnetic field is ZERO everywhere inside the coax outer conductor where our sensing wire is located. The apparatus is static and there is no mechanical motion or flux-cutting. Note that the physical position of the center wire is irrelevant so long as it’s inside the outer tube.

[11] Jefimenko, Oleg. D., op. cit. p. 31.

The situation is as shown below:[12]

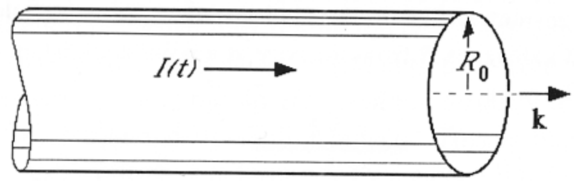

Fig. 3.2 A cylinder carries a time-variable current. There is an electrokinetic field outside and inside the cylinder.

Fig. 2. Induced Electrokinetic E field from changing currents in a cylinder.

In order to make this problem simple we will use a couple of tricks. The first is that $\mathbf{E_k}$ just outside the cylinder is equal to $\mathbf{E_k}$ just inside the cylinder, which is very thin. Next we use the common trick, which notes that the field outside the cylinder is identical to the case where the current in the outer tube is replaced with an identical current in the center wire. Thus we need only find the value of $\mathbf{E_k}$ R_o away from the center of a fine wire. That value will equal the value just inside the shield. And finally we note that by Ampere's law, symmetry and the fact that the center wire carries no current, that the magnetic field inside the tube is zero. This does not necessarily mean that **A** is zero, however, but it does imply that **A** has a constant value everywhere inside the tube. Hence, the value in the center is equal to the value at the edge.

We start with the electrokinetic field calculation found at the center of a wire of length 2L at distance R_o. Note that since the source current and the sensing wire are both in the same direction (z axis) no vector directions need to be resolved. The wire calculation is shown as:

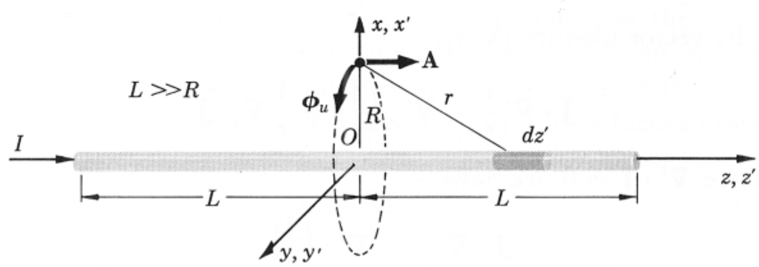

FIG. 11.1 Calculation of the magnetic vector potential associated with a segment of a current-carrying wire.

Fig. 3. Calculating Magnetic Vector Potential to Find Electrokinetic E.

[12] Jefimenko, Olg. D., "Electricity and Magnetism", Appleton-Century-Crofts, New York, 1966, p366-367.

The distance R from each current element to the observation point at the center of the wire at distance R_o from the wire is given by the hypotenuse of a right triangle as seen below.

Since

$$\vec{E} = -\frac{\partial \vec{A}}{\partial t}$$

The induced electrokinetic field is given by:

$$\vec{E}_k = -\frac{dI}{dt}\frac{\mu_o}{4\pi}\int_{-L}^{L}\frac{dz'}{\sqrt{z'^2 + R^2}} = -\frac{dI}{dt}\frac{2\mu_o}{4\pi}\int_{0}^{L}\frac{dz'}{\sqrt{z'^2 + R^2}}$$

And since we've given a rising current at 1 Ampere per second this means that the above minus sign that represents Lentz' Law, will give electric fields that oppose the rising current or in the minus x direction:

$$\vec{E}_k = -\frac{dI}{dt}\frac{2\mu_o}{4\pi}\ln\left(z' + \sqrt{z'^2 + R^2}\right)\Big|_0^L$$

$$\vec{E}_k = -\frac{dI}{dt}\frac{2\mu_o}{4\pi}\ln\left(\frac{L + \sqrt{L^2 + R^2}}{R}\right) = -2x10^{-7}\ln\left(\frac{2L}{R}\right)\frac{dI}{dt}$$

Where we have neglected R^2 compared to L^2 and can use this value for the electrokinetic field at the center of the wire and where L represents *half* the length of the coax rather than it's full length because of the way it was defined in our above derivation. Of course the induced field along the wire is not constant but falls off at the ends but since we are ignoring "end effects" for the moment, we can simply assume the center value applies along the entire length of the center wire. This makes the integration of the field along that sensing wire merely a multiplication by the length of the wire (1 meter). This then gives an EMF magnitude:

$$EMF = (2\,x.5)\,2x10^{-7}\ln\left(\frac{2x.5}{.002}\right) = 1.2429\ microvolts\ \frac{dI}{dt}$$

If the coax is 100 meters long, the calculation becomes:

$$EMF = 100x2x10^{-7}\ln\left(\frac{2x50}{.002}\right) = 216.396\ microvolts\ \frac{dI}{dt}$$

Since we have chosen dI/dt to equal a rise of one ampere per second, dI/dt = 1 so the above microvolts represent the measured induced voltages. If the current rises twice as fast then twice the voltage value is induced on the wire etc.

If a sinusoidal current drives the system as is commonly the case, where I = sin ωt and thus:

$$\frac{dI}{dt} = \omega \cos \omega t = 2\pi\ f \cos \omega t$$

Thus:

$$\vec{E}_k = 2\pi\ f \frac{2\mu_o}{4\pi} \ln\left(\frac{2L}{R_o}\right) \cos \omega t = \mu_o f \ln\left(\frac{2L}{R}\right) \cos \omega t = 4\pi f\ x 10^{-7} \ln\left(\frac{2L}{R}\right) \cos \omega t$$

or we cam find a sinusoidal $\mathbf{E_K}$ with an amplitude given by:

$$\vec{E}_K [Peak\ \ value] = 1.26 \ln\left(\frac{2L}{R}\right) f\ \ (in\quad megahertz)$$

which for our one meter long, 4 mm diameter coax above driven by a 1 megahertz current gives the peak value of:

$$EMF = 1.26 \ln \frac{2x.5}{.002} x1 = 7.83\ volts$$

Although we neglected "end effects" it would be instructive to calculate just for fun how the induced $\mathbf{E_k}$ electric field varies along the length of the sensing wire. To do this we start with Jefimenko's method above, but redefine the current as going from 0 to L rather than his choice of –L to +L as in this case the two integrations will be unequal and one side cannot simply be doubled.

It can be shown that the formula in this case is given by:

$$\vec{E}_k = -10^{-7} \left[\ln\left(\frac{X + \sqrt{X^2 + R_o^2}}{R_o} \right) + \ln\left(\frac{(L - X) + \sqrt{(L - X)^2 + R_o^2}}{R_o} \right) \right]$$

Where the first term represents the integral from 0 to the chosen measurement position X and the second term is the remaining integral from the position X to the end of the wire L. X in this equation is the position along the sensing wire where one is measuring the induced field that takes a value from 0 to L. L in this formula represents the entire length of the Coax as opposed to the half value of L in the above calculation. The final plot shows the variation of electric field along the sensing wire for a 1-meter coax. It is seen that the field is rather flat over a large percentage dropping off quite rapidly at the ends.

While it is tempting to assume that the falling induction is asymptotic to the ends of the wire, that is actually not the case. The electokinetic $\mathbf{E_k}$ field has finite values beyond the ends of the coax where there may or may not be wire where a voltage can be induced by the field. This can be thought of as "end effect" field "leakage" that is actually calculated by this method. This problem is difficult to set up using Faraday's law since the choice of surface upon which to measure the magnetic flux is not at all obvious. And even if one were to determine the surface giving the correct results, that law does not deal with the end effects.

This correct method of calculating the induced field not only does not require the closed loops of the Faraday equation, but also, as is seen, can actually calculate the distribution of inductance within a system of conductors. Today, such calculations have become of increasing importance, as micro miniature integrated circuits require greater knowledge of the current and heat distributions in microscopic traces.

http://www.hypersphere.us/Faraday.pdf

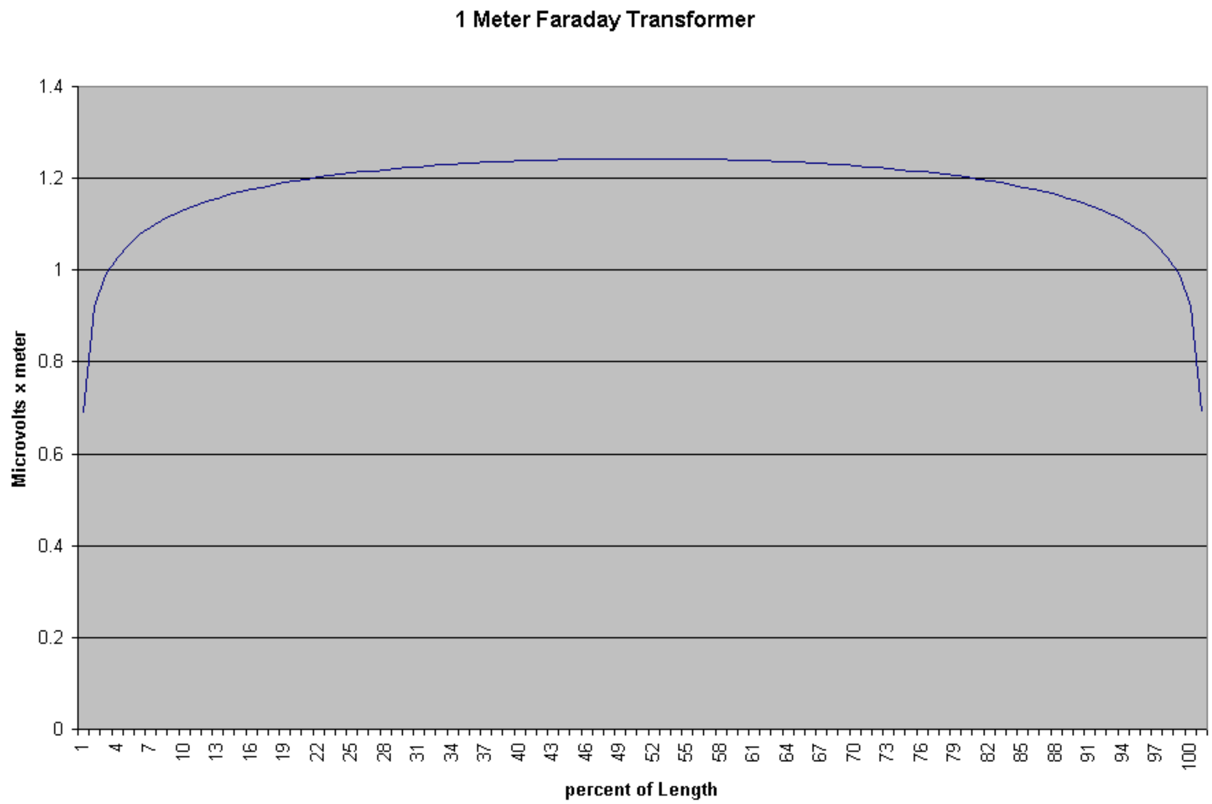

Fig. 4. Distribution of voltage induced along the center conductor of a 1 meter length of coax 4mm in diameter when current in the outer shield rises at one ampere per second

Correction: Note that the abscissa in the above plot has been labeled incorrectly in that the extreme values of the above plot of 0.691 microvolts occur at 0 and 100 percent and not at the values that seem to be shown by the plotting software.

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(version 0.4)