Title Page (version 0.2)

The Peng Kuan Paradox

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The Peng Kuan Paradox (ver. 0.2)

Peng Kuan has a website[1] in which he investigated various electromagnetic subjects. The Peng Kuan Paradox is one of these and basically involves the conditions for induction of voltages and currents. Given some source of induction such as a long solenoid or toroid coil, if a circular conductive wire loop which has a thin cut in it is placed about that source and an increasing current is applied to the source coil, Faraday's law states that

$$EMF = -\frac{d\,\Phi}{dt}$$

Where Φ is the magnetic field integrated over the area of the coil and called the magnetic flux. Electromotive force termed here EMF is rather undefined at this point but is said to mean the "force" which creates an induced current in the loop were it a complete circle. In a practical case if we were to put a voltmeter across the gap in the conductive loop, what it reads would be related to our undefined EMF. A rising or falling magnetic flux would be related to the appearance of a voltage at the gap. This is the essence of transformer action.

While Faraday's Law does not deal with electric fields occurring at our loop (He was more into trying to make sense of "lines of force" whatever they might be) it can be shown from Maxwell's equations that an electric field related to the rising magnetic flux does indeed occur at the wire. Since the wire is conductive, this electric field applies a force to the free charges. Were the circuit complete it would cause a current to flow and hence the electric field would create the forces F = qE that cause the charges to circulate in a current.

But when the conductor is cut, charges cannot circulate. What happens is they build up on one side of the cut leaving opposite charges on the other side of the cut (in metallic conductors negative charges are free to move and positive charges are fixed to the material). This displacement of charges, creates an electrostatic field. E_S in the wire. This field must exactly cancel the induced electric field that we will term after Jefimenko[2] an *Electokinetic Field* and he notes that the force produced by this field on charges to create a current is properly termed the EMF. The reason these electric fields must cancel is that at equilibrium charges in the conductor are not moving and hence experience no acceleration and hence no force.

The crux of the Peng Kuan paradox is that if one then adds these two electric fields together the total field inside the wire is *zero* and thus a line integral E · dl around the wire from one side of the gap to the other must be zero because the total electric field is zero!

Note that were there no cut in the circle of wire, charges experiencing the force from the induced electrokinetic E_k electric field would simply circulate in the loop. There would be no electrostatic field because there are equal numbers of positive and negative charges in the wire and they remain evenly distributed if allowed to move. Hence in this case the integral of $E_K \cdot dl$ around the wire one time would give a value read on a voltmeter and it would be discovered that the current in the wire would be given by that reading divided by the resistance of the loop which is to say by Ohm's law.

1 http://www.pengkuanem.blogspot.com/

2 Jefimenko, Oleg D., "Causality Electromagnetic Induction and Gravitation", Electret Scientific Co. Star City, 2nd ed. 2000, pp 28-29.

The only oddity is that the induced electric field E_K is not an electrostatic field, $\mathbf{E_s}$ and has a number of different properties from one. It is not a conservative field so that means the value of the line integral will depend upon the path taken. If the path loops around the source coil twice, twice the voltage will be induced into the loop. Indeed if V is the voltage induced in one turn, nV will be the voltage measured as induced into n turns. There is no paradox in this case because there is no electrostatic field and we are simply calculating the induced field, $\mathbf{E_K}$ over some closed path to get the voltage.

Strange things happening with induction fields was demonstrated by MIT professor Emeritus Walter Lewin in which he showed the puzzling case where two voltmeters connected to the same points in a circuit have two different reading, in fact one reads positive and the other negative in spite of both meters being connected with the same polarity.[3] This created some discussion with even some professors viewing the results with disbelief. A paper examining such situations has been written and may have been the source of the demonstration.[4]

In spite of the extensive application of Maxwell's equations for more than a century, the confusion still created by these issues indicates that there have been a few things swept under the electromagnetic rug. Some of these issues will now be examined here.

The One E Field Dogma

The "one E field dogma" largely promoted in the 1940s by J. Slepian,[5] stated that there is "only one E field". This is a rather odd conclusion given that one can identify electric fields of various types with an amazing array of differing properties. For example there is an electrostatic Electric field produced by stationary charges. Such a field is irrotational, is conservative (all closed path line integrals of the field are zero and all line integrals between two points give the same value no matter what path is chosen), is radial about point charges, follows an inverse square law, and the field is distorted by charges brought into it. On the other hand, an electrokinetic electric field produced by induction is solenoidal, is non-conservative (closed line integrals are often path dependent and do not always equal zero), forms closed loops, does not follow an inverse square law, and the field is not distorted by bringing in charges. One can even note that there is a third type of electric field which is a **v x B** "Lorentz field" which results in forces on charges moving in a magnetic field. Such a field is even more strange in that it only exists at the charge and depends upon a second field.

Of course there are those today who argue that electric and magnetic fields (as different in characteristics as they are) are just one field since motion can transmute one into the other. While such mathematics may be possible, it hardly creates a convenient system of practical calculation and understanding. We make the same argument here for dividing E fields up into pieces. However, we need to note that Jefimenko has shown that the so-called Lorentz **v x B** electric field falls out of the mathematics of the Electrokinetic electric field in a natural way simply when one gives a motion to currents.[6] Thus we need only divide electric fields into just two fields rather than three.

3 Walter Lewin: https://www.youtube.com/watch?v=FUUMCT7FjaI

4 Romer, Robert H., "What do Voltmeters measure?: Faraday's law in a multiply connected region", Am. J, Phys., Vol 50, No. 12, Dec. 1982, pp1089-1093. http://www.uvm.edu/~dahammon/Demonstrations/5ElectricityAndMagnetism/5bElectricFieldsAndPotential/5b10ElectricField/Faraday%27sTeaser/Romer/Romer.pdf

5 Joseph Slepian, former Associate Director, Research Laboratories, Westinghouse Electric Corporation, East Pittsburgh Pa.

6 Jefimenko, Op. Cit. P33 ff.

Romer comments on such a division in another paper saying:

"In Shadowitz's initial discussion of Faraday's Law, he makes an artificial division of the electric field into conservative and non-conservative parts, a distinction not likely to be respected by real meters which respond to the total field."

The fact as we have already seen in the Peng Kuan paradox, a case where the total field is obviously zero and yet the meter is reading a voltage, makes the above statement suspect. Furthermore, we would not agree that a separation of an E field into static and kinetic parts is particularly "unnatural", because the static part is irrotational and the kinetic part is solenoidal, and it has been noted:

"All vector fields will be found to to be made up of one or both of two fundamental types: Solenoidal fields that have identically zero divergence everywhere and irrotational fields that have zero curl everywhere. The most general vector field will have both a nonzero divergence and a nonzero curl. We shall show that this field can always be considered as the sum of a solenoidal and an irrotational field. This statement is essentially the content of Helmholtz's theorem." [7]

Thus, it is argued that this is indeed a "natural" division with the sum of both fields being the general electric field. It is important to note that all the of "electric fields" we discussed above are force fields and thus produce forces on charges F = qE regardless of the field type. For this reason, when calculating the forces on charges, these fields of differing properties can simply be added up as we did inside the conducting wire. The conclusion that the total field inside the wire loop is zero is therefore valid. The total force on a charge q due to a general electric field is thus given by:

$$\vec{\boldsymbol{F}} = q(\boldsymbol{E}_{ELECTROSTATIC} + \boldsymbol{E}_{ELECTROKINETIC} + \boldsymbol{E}_{LORENTZ}) = q(\boldsymbol{E}_s + \boldsymbol{E}_k + \boldsymbol{v} \; x \; \boldsymbol{B})$$

However, because the nature of these fields is so different other operations may not be allowed to be exchanged from one to the other. Care must be exercised.

Causality and True Sources

Causality is a natural law in the Earth. It is the often forgotten rule that all present phenomena are determined only by past events. Since according to relativity, energy and hence information cannot travel faster than the speed of light in vacuum, it directly follows that two simultaneous events separated by any distance cannot be the cause of each other. This is the fact that causes the 19^{th} century concepts of "action at a distance" to be rejected as bogus (until recently, anyway). Simultaneous events not separated by a distance are more troublesome, but in the case that the relationship is a time derivative such as Faraday's law or Maxwell's equations, we observe that a derivative by definition cannot be calculated from a single point from a given function. Values of the function from the past and future are needed and a limit taken down to the single point. This implies that such equations are not causal relationships either.

But it is a wide practice in physics (and science and technology in general) when looking at an equation to automatically assume that one side of the equation "causes" the other, when in fact the relationship is only that one side is *equal* to the other. One, hears, for example that E and H fields "cause each other" or that a changing magnetic flux through a loop "causes" an EMF and a current to flow and oddly this

7 Plonsey, Robert, and Robert E. Collin, Prionciples and applications of Electromagnetic Fields, McGraw-Hill, New York, 1961, p. 29.

occurs even when, as in the case of a toroid, that there is no magnetic field at the location where the EMF is produced. The importance of causality will be seen because electromagnetics is rife with redundancies. There are a great many examples of situations where there are various ways to calculate a value, all giving the “correct answer”, but differing wildly in philosophical implications. The case of Faraday's law being one such situation. That law as given above is assumed to say that a changing magnetic flux induces an electromotive “force” which we presume is an electric field that moves charge creating a current. This is said to occur even when there is no magnetic field near current but only passing through the center of the current loop. This is clearly a case of “action at a distance”. Thus, the true source must lie elsewhere.

Jefimenko has examined Maxwell's equations for causality and derived a set of equations for E and H[8] that he suggests indeed are causal. From these equations we observe what is purported to be the true source of the EMF and that source is the time rate of change of a current. Oddly this is exactly how Faraday himself described the actions that later became his “law”:[9]

“*When an electric current is passed through one of two parallel wires it causes at first a current in the same direction through the other, but this induced current does not last a moment notwithstanding the inducing current (from the Voltaic battery) is continued... but when the current is stopped then a return current occurs in the wire under induction of about the same intensity and momentary duration, but in the opposite direction to that first found.*”

The situation is explained by Jefimenko that a changing current creates an electric field of some type about itself according to the equation:

$$\vec{\boldsymbol{E}}_{\boldsymbol{k}} = -\frac{1}{4\pi\epsilon_o c^2}\int \frac{1}{r}\left[\frac{\partial \bar{\boldsymbol{J}}}{\partial t}\right] dv'$$

In this equation the Field $\mathbf{E_K}$ he terms the “electrokinetic field”, the square brackets indicate that current density is “retarded” which is to say that allowance must be made for the time it takes for the influence to travel from the current to the electric field observation point, and that r is the direct distance from the observation point to the current element being integrated. Note that the electrokinetic field exists everywhere about each current element and is parallel or anti-parallel to the direction of the current. Thus, this field also exists inside the wire carrying the source current and fills all space out as far as the field has been able to travel at the speed of light since the time when the current began to change.

The minus sign represents Lentz' law as when the source current in increasing an electrokinetic field is induced inside the wire that opposes the flowing current, whereas when the current is decreasing, the electrokinetic field inside the wire is in a direction that produces forces on charges that tend to keep them moving rather than stopping. This is the basis of self-inductance. Note that once again, in the case of the changing magnetic flux philosophy, in straight wire bearing a current the Biot-Savart law shows no magnetic field down the axis of the wire, whereas the electrokinetic electric field is everywhere including down the axis of the wire. This is an important observation given that straight wires are known to have inductance.

While there is no magnetic field outside a toroid, there is something out there of interest. That

8 Jefimenko, Op. Cit. See Chapter one.
9 Faraday in a letter to Richard Phillips, November 29, 1831.

something would be the magnetic vector potential, **A**. This exists even though the magnetic field may be zero. This represents another redundancy in electromagnetics. If we remember that **A** is given by the equation:

$$\vec{\boldsymbol{A}}=\frac{1}{4\pi\epsilon_o c^2}\int\frac{\vec{\boldsymbol{J}}}{r}dv'$$

And therefore we can observe that:

$$\vec{\boldsymbol{E}_{\boldsymbol{K}}}=-\frac{\partial[\boldsymbol{A}]}{\partial t}$$

Again we note that the magnetic vector potential, **A**, also takes time to travel from the source current to the observation point and thus, is also retarded. One might also conclude that **A** might be the source of the electrokinetic $\mathbf{E_K}$ field, but the fact that both fields are simultaneous and the time derivative disproves that idea. The bottom line, here is that if one has a changing current in space, there is thrown out from it at the speed of light, an electrokinetic $\mathbf{E_K}$ field, a magnetic field (which spread over an area would be a magnetic flux) and a magnetic vector potential field, **A**, all three of which are delayed by transmission time from the source current density **J**. However they are all three related and hence using of any one of them in a calculation can give a "correct" answer.

Line Integrals of Electric Fields

Electric fields of whatever type are force fields where the force, **F,** on a "test charge" q, is given by q**E**, where **F** and **E** are vectors in the same direction and **E** is an electric field of the Electrostatic, Electrokinetic or even Lorentz (v x B) type. So it is of interest to ask how much work does it take to move a test charge q from point A to point B within an electric field of *any* type? We move this test charge without allowing it to accelerate so we must apply an external force to the charge equal to the electric field at every point such that the force on the charge is always zero as the charge moves over our chosen path between A and B. Work is force times distance and if you take into account the angle between the force generated by the electric field and the direction of our chosen path, we obtain the mathematical line integral for the work $W_{AB.}$

$$\text{Work moving q from A to B}=W_{AB}=q\int_A^B\vec{\boldsymbol{E}}\cdot d\vec{\boldsymbol{l}}$$

From this we can now define something termed "voltage difference" as:

$$\text{Voltage difference}=\frac{W_{AB}}{q}=\int_A^B\vec{\boldsymbol{E}}\cdot d\vec{\boldsymbol{l}}$$

Since the electric fields we have described have different characteristics, so does the nature of this integral and "voltage difference".

The electrostatic field from a stationary charge distribution is especially interesting in that such a field is termed "irrotational" which means it never forms loops and the force "lines" are all radial from the charges. And this leads to the interesting fact the value of the voltage above does not depend upon the path chosen but only on the location of the end points. This is termed a conservative field and has the property that the above integral about *any* closed path is always zero. In addition it is found that if one has measured the voltage difference between some reference point (which may or may not be far away and termed "infinity" in mathematics) and all the other points in space where the field exists, one finds that from just those voltage values one can calculate what the electrostatic field was that produced those voltage values. In other words this field can be derived from a scalar potential.

Voltage is defined the same way for an electrokinetic field but in that case the field does indeed form loops, so is termed "solenoidal" and the integral around such loops is not necessarily zero and the value of the voltage integral very much depends upon the path one chooses to follow. The interesting point from an electrical engineering viewpoint is that because the work from moving charge around a closed loop in an electrostatic field is zero, such a field cannot provide a continuous current around a "circuit". A capacitor can supply charge into a resistor to heat it, but eventually the charges on the plates are not being replaced and are lost and the current stops. A non-conservative field, on the other hand because the work integral around a circuit is not zero can supply energy to the resistor (dissipated as heat) with every trip around the loop.

For historical reasons and because a non-conservative solenoidal field can move charges around a closed loop by means of the forces of the E field the voltage calculation in this case is often termed an *electromotive force* or *EMF.* Such terminology reminds the observer that this electric field can move charge continuously in a closed circuit.

Because we are calculating the work needed to move a test charge, one can invoke the work-energy theorem which states that the work done on a particle by the resultant force is always equal to the change in kinetic energy of the particle. Because of this the Peng Huan situation of an electrokinetic field around a ring where the ring of wire is replaced with an evacuated tube filled with electrons represents a particle accelerator known as a *betatron.* Every time an electron goes around the tube it represents a certain kinetic energy added to the particle and a very high velocity and hence kinetic energy can be obtained. Note that electrostatic field can also accelerate charged particles like electrons but since the energy integral above is zero around a closed loop, these devices are arranged as long straight tubes with increasing voltages down the tube. These are termed linear accelerators.

The Peng Kuan Integral

In the case of the zero current situation described above, we have noted that there are two electric fields present. There is the electrokinetic field from the central current source and the electrostatic field generated by the charge displacement in the wire. The total field inside the wire is therefore:

$$\overrightarrow{E_{total}} = \vec{E}_k + \vec{E}_s = 0$$

This field must be zero because there is no current in the wire and thus, no forces on charges. If we now wish to computer the above line integrals we see that these integrals consist of two parts. If we label one side of the gap in the wire as A and the other as B, the integrals consist of the integral in the wire from A to B plus the integral across the gap from B to A. Two things we already know is that the total line integral around the entire loop for the electrostatic part is equal to zero and the total integral around the loop for the electrokinetic part is equal to some voltage V. Inside the wire the line integral is given by:

$$\int_A^B \overrightarrow{E_{total}} \cdot d\vec{l} = \int_A^B \vec{E}_k \cdot d\vec{l} + \int_A^B \vec{E}_s \cdot d\vec{l}$$

In the gap the integrals are given by:

$$\int_B^A \overrightarrow{\boldsymbol{E}_{total}} \cdot d\vec{\boldsymbol{l}} = \int_B^A \overrightarrow{\boldsymbol{E}_k} \cdot d\vec{\boldsymbol{l}} + \int_B^A \overrightarrow{\boldsymbol{E}_s} \cdot d\vec{\boldsymbol{l}}$$

Since the gap is small the integral across the gap for the electrokinetic field can be neglected. Thus, the total integral around the entire loop is given by:

$$\oint_A^A \overrightarrow{\boldsymbol{E}_{total}} \cdot d\vec{\boldsymbol{l}} = \oint_A^A \overrightarrow{\boldsymbol{E}_k} \cdot d\vec{\boldsymbol{l}} + \int_A^B \overrightarrow{\boldsymbol{E}_s} \cdot d\vec{\boldsymbol{l}} - \int_A^B \overrightarrow{\boldsymbol{E}_s} \cdot d\vec{\boldsymbol{l}} = V + 0$$

And lastly because we know the electrostatic line integral around the loop is zero this implies that the line integral of the electrostatic field inside the wire is equal and opposite to the line integral across the gap. In other words:

$$\int_A^B \overrightarrow{\boldsymbol{E}_s} \cdot d\vec{\boldsymbol{l}} = -\int_B^A \overrightarrow{\boldsymbol{E}_s} \cdot d\vec{\boldsymbol{l}}$$

The Voltage from A to B clearly equals the induced value V and *also* equals 0, which obviously needs further discussion!

However, if we were to put our solenoid and split loop away in a corner somewhere and run two wires from each side of the loop gap out to a meter there would be no problem. The external field would be due solely to charges and hence would be conservative. And this thus forms the basis of the circuit element known as a transformer. Namely that a changing current in one coil of wire creates a voltage at the terminals of a second coil of wire. The fact that one coil can be insulated from the other coil forms part of the utility of this device and another feature is that that since the secondary voltage is n times that of a single turn where n is the number of turns around the flux, it is possible to step voltages and currents up and down by choice of how many turns the various coils contain.

What We Can Learn from Faraday.

But before discussing the nature of circuit theory let us first discuss Faraday's law. We have seen that this relationship between a changing magnetic flux and an EMF is not a causal one as is widely believed. We can observe that Feynman in his famous textbooks when discussing circuit elements constantly assumes that they produce no magnetic field external to their enclosures.[10] From this he assumes that all electric fields are conservative and hence only electrostatic. But this assumption is based on the error that a changing magnetic field "creates" a non-conservative electric field and if you have no magnetic field you can have no non-conservative fields. But we have seen that in truth it is a changing current that creates a non-conservative electrokinetic E field and this field exists in all space around the current source. Agreed that it falls off inversely with distance, but it clearly is NOT confined to an area of space the way the magnetic field inside a toroid can be confined. Hence a solenoid or toroid does NOT confine the electrokinetic fields the way the magnetic fields are confined!

So then one must ask just how can the circuit approximation of only conservative fields work if there are non-conservative fields in the region? To answer that one must remember that Faraday's law is a true equality. The fact that magnetic fields do not created electric fields does not mean that the two are not related by an equality. Therefore Faraday's law can tell us things about an electrokinetic non-conservative electric field. And a further point is that while a calculation of voltage (line integral) using non-conservative fields may depend upon the path you choose, it is not necessary that every path produce a different value.

10 Feynman, Leighton, Sands, "Lectures on Physics", Addison-Wesley publishing Co., Palo Alto, 1964, Vol II, Section 22-3

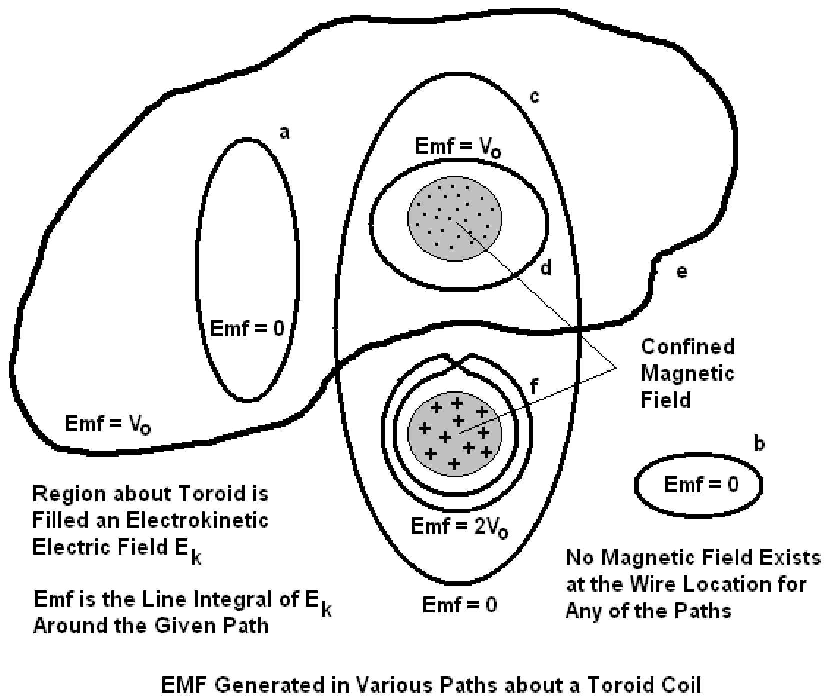

Fig. 1 Faraday induction into various paths about a toroid coil.

A number of arbitrary paths about a toroidal winding are shown in **Fig 1**. By Faraday's law we know that the EMF calculated using any given path is equal to the amount of magnetic flux enclosed in that path. Hence if any path is chosen outside the toroid where there is no magnetic field, as in loops a and b in spite of the non-conservative electrokinetic field that may exist there (even strongly), there is zero EMF produced in the loop, which is to say the line integral about those loops is zero. Thus, *any* closed path not enclosing a magnetic flux must produce a line integral (voltage) of zero just like a conservative electrostatic field. Thus, by confining magnetic fields to a region one creates a conservative situation outside that region justifying Feynman's "no magnetic fields" assumption. But this is no universal escape as Feynman implies. The fact that non-conservative electrokinetic fields are still present can lead to effects such as the Lewin demonstration or the Peng Kuan paradox above where different voltages are measured with meters at the SAME two points depending on the path taken *even when the voltmeter lead loop seems to contain no changing magnetic flux!*

There is more to learn as well. It is clear if our path circles the flux in our toroid, but does so as in loop c where flux in one side of the toroid is in one direction and the flux in the other side is in the other direction and the path includes them both, then they cancel producing a zero result. Hence even though there is flux through our loop the flux in one area cancels that in a completely different location giving no induction. The standard thing is a loop, such as d, though the toroid hole circling its internal magnetic field and thus giving a line integral equal to the induction value of EMF. If this path goes around the flux twice the EMF is doubled and if it goes around n times the resultant EMF is n times greater. Another thing that Faraday tells us is that in feeding a wire loop around the toroid, so long as we feed the wire through the hole in the toroid, the path of the wire does not matter as shown by the path e. So again we see a quasi-conservative effect with a non-conservative field. Lewin demonstrates this effect in his video. But path does indeed matter and in path f the wire circles the toroid twice and the induced EMF is doubled. Romer[11] who finds a discussion of conservative vs. non-conservative fields "unnecessary", does note this quasi-path-independence of line integrals calling them "pseudo-conservative". We note that it is this "pseudo-conservative" feature that allow so many circuit concepts to be blindly applied to inductive non-conservative situations and is just one more example of the widespread redundancy found in Electromagnetics.

Circuit theory

Generally speaking one cannot take a device with hundreds or thousands of parts such as a radio or computer and calculate it's operation using Maxwell's equations. There is just too much complexity. However if one can restrict operation in certain ways creating a conservative situation, things become much simpler and ordinary algebra can be used to obtain answers.

Such a conversion from non-conservative fields where the line integral is non-zero and depends on the path to a conservative version where the path does not matter and the line integral around such a loop is always zero is seen above in our coil and loop transformer device. If wires are run from the gap in our loop where charges have built up, only electrostatic conservative fields exist away from the device. In order to make circuit theory work it is necessary to keep all electric fields electrostatic or at least such that path does not matter so that a unique voltage difference will be measured between any two points regardless of the path taken to measure it and that the current into a device equals the current out of it. For these reasons frequencies must be kept low enough that there is no radiation losses and things like inductors and transformers designed so that external electrokinetic fields are at least not a problem if not actually eliminated. Wires are usually assumed to have zero resistance and practical real devices are usual modeled as "ideal" which is to say the inductance of capacitor leads is ignored and the capacitance between turns on an inductor coil is ignored. In many cases these approximations work quite well giving results with reasonable accuracy.

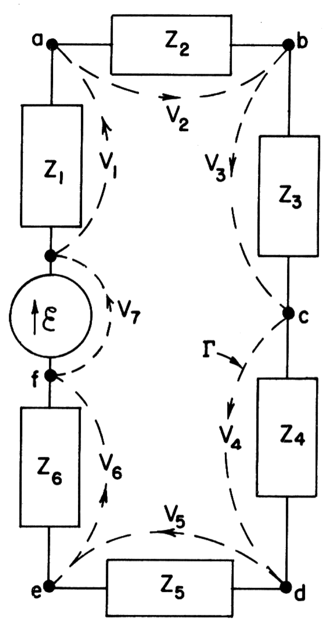

Fig. 22–9. The sum of the voltage drops around any closed path is zero.

Fig 2. Electrostatic line integral in a circuit approximation (Feynman Fig. 22-9)

11 Romer, Robert H., Op. Cit., p. 1091

Kirchoffs laws (or “rules” as they are sometimes termed since they are based upon approximations) say that the sum of currents into a given point must be zero, and the sum of voltages around a circuit must also be zero. Or the second rule stated a different way would be that the voltage *rises* of sources must equal the voltage *drops* in the passive components (resistors, capacitors, inductors). The latter is easily proved by choosing path outside our components as shown in Feynman's figure (**Fig.2**). Since we have created a situation where all fields outside the component are conservative, that line integral must be zero proving the rule. Given these approximations then simple algebra can solve the most complex field theory problems and this is what much of certain areas of electrical engineering is about.

A closer look at the paradox

To return to our solenoid and loop and gap above, it now becomes necessary to explain how a reading from the loop side reads zero on a meter, while one across the gap in the loop reads V even though the meter circuit path encloses no flux in either case. The situation is shown in **Fig 3a**. We assume the “gap” is air or a very high value resistor that we are measuring the voltage across (it actually doesn't have to be a high resistance value).

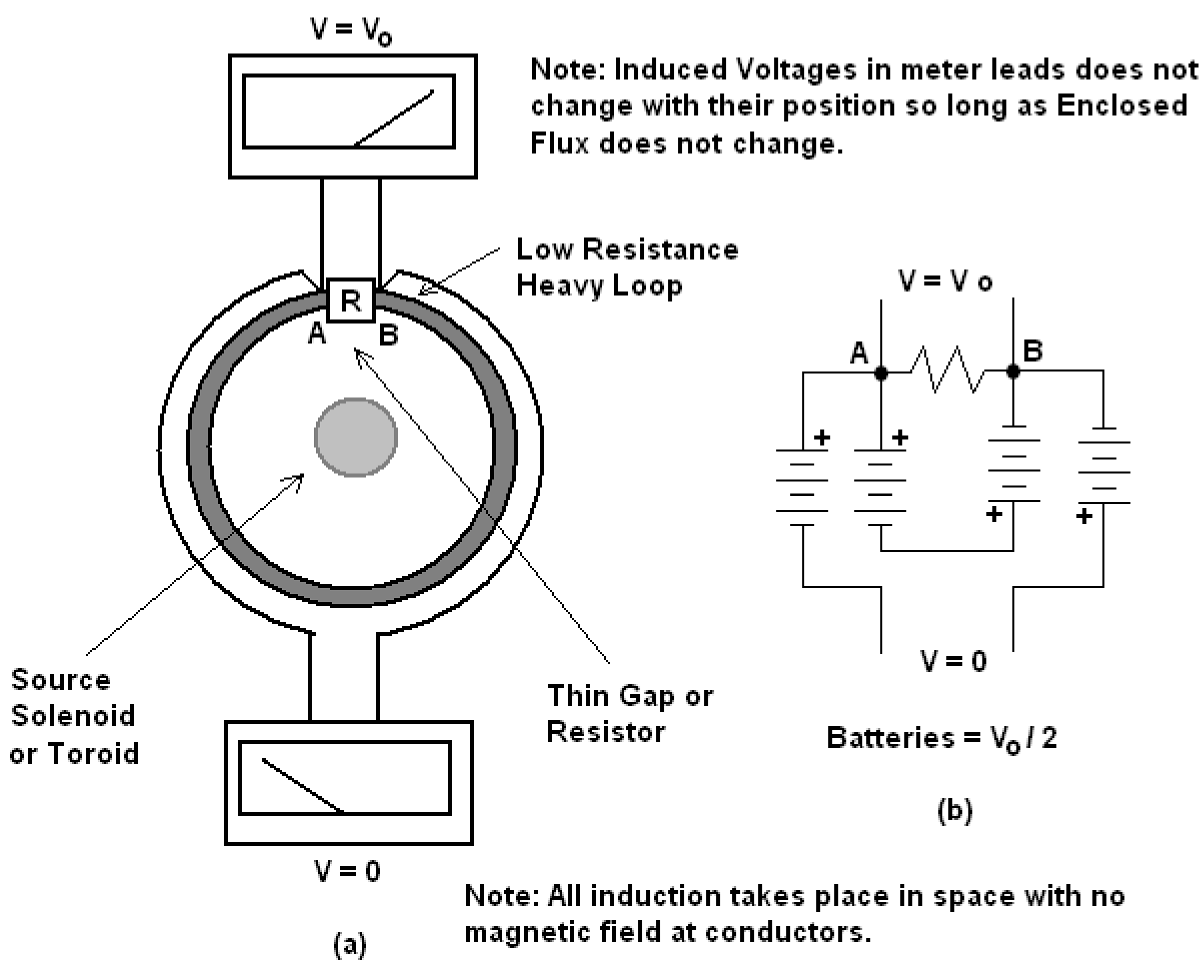

Fig. 3 Situation of voltage induced in conductive loop with thin gap to prevent current flow.

An understanding can be gained by noting that the wire leads of the meter on the loop side are parallel to the main loop and thus are in virtually the same electrokinetic field as the original loop. For this reason voltages are being induced into the meter test leads that oppose the potential from the static build-up at A-B. If we replace the induced voltages with batteries as shown in **Fig. 3b**, the situation becomes more clear. Thus, two meters apparently connected to the same points, A and B seem to give entirely different readings!

Another remark is that by what we learned from Faraday above, it turns out that we don't need to lay our meter leads close to the induction loop at all. So long as we don't enclose any flux we can bring them to A and B by any path we choose creating the illusion in our minds that there are no non-conservative fields in the area. Which is obviously not true. And the Romer quote above suggesting that “real” meters will not measure the oddities of non-conservative fields in clearly misguided.

A final note would be to look at the situation from the point of view of Faraday. If we assume no resistor in the gap in the loop and that all voltmeters have an “infinite” resistance, it is clear that the path of the wires connected to the meter reading zero, have enclosed no flux from the source. On the other hand, the path of the meter reading V (especially if there is no resistor in the gap) is clearly seen to circle the source flux by reason of the path through the heavy conducting loop.

The bottom line here is that while circuit theory completely fails in this case (electric fields are not limited to Electrostatic $\mathbf{E}_s$ fields) and two chosen points appear to be able to give two entirely different voltage readings, the physics of the situation has not failed and results are completely reasonable.

What About a Resistor in the Gap?

It have been noted in the above example that a good low resistance conductor actually converts a non-conservative $\mathbf{E_K}$ field to a conservative $\mathbf{E_S}$ field by rearranging the charges inside the conductor. For this reason a transformer becomes a good circuit element. All electric fields external to the transformer are either electrostatic and conservative or they act as if they were conservative being independent of path.

We have also mentioned the opposite situation of a resistor in the shape of a ring placed around solenoid or toroid source. Since there is no gap, in this case a current flows and because a current flows, there is no charge build-up creating electrostatic fields. Hence, the electric field inside the resistor is completely electrokinetic. The key relation that has been experimentally found for resistive materials is given by:

$$\vec{J} = \sigma \vec{E}$$

Here **E** will equal $\mathbf{E_K}$ as there are no displaced charges and **J** is current density. Here σ is a material property called the conductivity of the medium in mhos per meter. The current flowing through a resistor is the integral of the current density over it's cross-sectional area or in the case where the E field can be considered uniform over the area is equal to **J**A where A is the area of a slice through the resistor. Hence:

$$I = \sigma \int \vec{E_k} \cdot d\vec{s} = \sigma A E_k$$

Where **E** is now the magnitude of $\mathbf{E_K}$ at the resistor. We know that the voltage around the loop, which is to say through the hoop resistor is equal to the line integral of $\mathbf{E_K}$ around the loop and by Faraday's law is equal to V_o or :

$$V=\oint \vec{E}_k \cdot d\vec{l} = E_k C = V_o$$

Where again E_K is the magnitude of $\mathbf{E_K}$ at the resistor and C is the average circumference around the resistor. If we now substitute E_K from the second equation into the first we obtain:

$$I=\frac{\sigma A V_o}{C}=(\frac{\sigma A}{C})V_o$$

Where the term in front of the voltage is called "conductance" but is usually expressed as it's inverse termed "resistance". This means that resistance increases linearly with the length of the loop and decreases inversely as the cross-sectional area increases. Hence:

$$I=\frac{V_o}{(\frac{C}{\sigma A})}=\frac{V_o}{R}$$

What we have found is that the current induced in this circular resistor purely by electrokinetic fields is equivalent to an external resistor driven by a voltage source equal to the EMF, V_o induced in any loop about the inductive source. Oddly we will find this rule true even if the resistor comprises only a fraction of the loop with the rest of the loop continued with low resistance wire. And even if the resistor is located within the induction electrokinetic $\mathbf{E_K}$ field that is inducing a field into it as well as the voltages applied to it's ends.

Suppose for example we divide our loop into two pieces. One piece is of the resistance material above of length r and has a resistance $\mathbf{R_o}$ and the other half is wire of length w and zero resistance. The total distance around the loop is it's circumference C as above. What happens now? This is shown in **Fig. 4**.

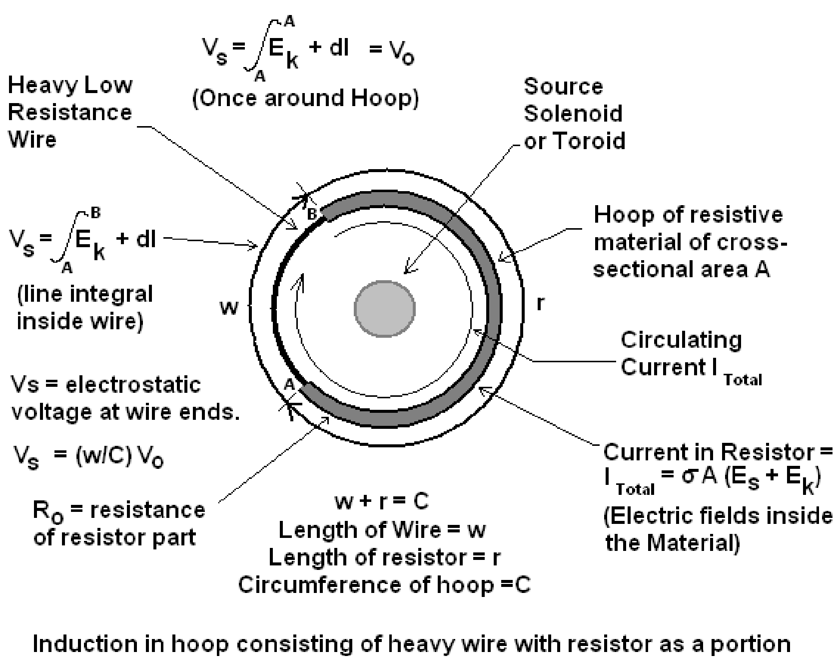

Fig 4. Situation with a hoop consisting of both wire and resistor material

We know that electric fields can be added so that:

$$J=\sigma(E_s+E_k)$$

and inside the resistor we find the current given by:

$$I=\sigma A E_s+\sigma A E_k$$

Where A as before is the cross-sectional area of the resistive material and σ is the conductivity of the material. Here we have an electrostatic electric field produced by the charges that accumulate on the ends of the wire part of the loop, and an electrokinetic field that is generated in the region by out source solenoid or toroid. Hence there are two effects producing current in the resistor.

To discuss how currents work in this situation one must recall how the induced electric field in the wire with a gap created a surplus of positive charge at one end and a deficit of charge at the other. The field throughout the wire had to be maintained at zero since no charges were moving and hence force on all charges had to be zero.

If, however, we insert a resistor in the gap it allows charge to be leaked off one end of the wire and travel through the resistor to the other end of the wire. This creates a field imbalance within the wire and charges will start to flow. Since the wire is highly conductive, it takes extremely little field to cause the charges to be moved (taking energy from the applied electrokinetic field) such that again equilibrium is established. A perfectly conducting wire would take virtually no internal electric field to create an "infinite" current. Thus, the current around the loop is determined only by the currents in the resistor portion!

We know from Faraday that a loop of circumference C around out source with a narrow gap will develop a potential between the ends of V_o which allows us to find the value of the electrokinetic field at our ring:

$$E_k=\frac{V_o}{C}$$

But in our case the wire doesn't go completely around the loop, but rather only a fraction w/C of the way so the potential between the ends of the wire that is applied to the resistor portion to create the electrostatic current is reduced from $\mathbf{V_o}$ by that amount so that from the integral of $\mathbf{E_k} \bullet \mathbf{dl}$ one finds:

$$V_s=E_k(w)=\frac{V_o}{C}(w)$$

The actual electrostatic field in the resistor is then given by the potential across it, $\mathbf{V_S}$ divided by the length of the resistor material, r, or :

$$\boldsymbol{E_S}=\frac{\boldsymbol{V_S}}{r}=\frac{\boldsymbol{V_o}}{C}\left(\frac{w}{r}\right)$$

Substituting this into our equation for current, we find the current due to the electrostatic field is given by:

$$I_1 = \frac{\sigma A}{C} V_o \left(\frac{w}{r}\right) = \frac{V_o}{\frac{r}{w}\frac{c}{\sigma A}} = \frac{V_o}{\left(\frac{r}{\sigma A}\right)}\left(\frac{w}{C}\right)$$

The denominator is recognized as simply the resistance of the material non-wire portion of our loop or R_o so that:

$$I_1 = \frac{V_o}{R_o}\left(\frac{w}{C}\right)$$

Thus we see as previously noted that if the wire comprises virtually all of the loop then the current through a load is determined by the total induced emf, V_o and the value of the load resistor following the relation of Ohm's law. On the other hand if the loop is ALL resistive material, then w = 0 and there is no contribution of electrostatic produced current. As we previously noted, in that case there are no electrostatic fields in the loop because there is no charge displacement.

This leaves us with the calculation of the current due to the electrokinetic electric field inside the resistive material:

$$I_2 = \sigma A E_K = \sigma A \left(\frac{V_o}{C}\right) = \frac{V_o\left(\frac{r}{C}\right)}{\left(\frac{C}{\sigma A}\right)\left(\frac{r}{C}\right)} = \frac{V_o}{R_o}\left(\frac{r}{C}\right)$$

Thus:

$$I_{TOTAL} = I_1 + I_2 = \frac{V_o}{R_o}\left(\frac{w}{C}\right) + \frac{V_o}{R_o}\left(\frac{r}{C}\right)$$

And since w + r = C the total circumference:

$$I_{TOTAL} = \frac{V_o}{R_o}\left(\frac{w+r}{C}\right) = \frac{V_o}{R_o}$$

Thus we find the interesting redundancy that if a resistor is inserted into a loop of wire receiving induction from some coil such as a toroid, the electrostatic contribution to the current and the contribution from direct induction into the material of the resistor from the source are divided by the ratio of the length of the wire to the length of the resistor. But that the TOTAL current in the loop always acts as if the induced EMF, $\mathbf{V_o}$ were a battery of that voltage feeding the resistor no matter if the resistor is within the inductive electric field or not.

Thus in the Lewin demonstration which will be discussed next, one need not ask “what about the induction into the resistors themselves by the solenoid. His “explanation” of simply replacing the induced emf with a battery of voltage $\mathbf{V_o}$ feeding the two resistors in series is totally valid.

The Lewin Paradox

As we indicated earlier Professor Emeritus Walter Lewin of MIT has preformed a demonstration of electromagnetic induction producing results that confounded students and even some faculty. Consider the Lewin arrangement shown below:

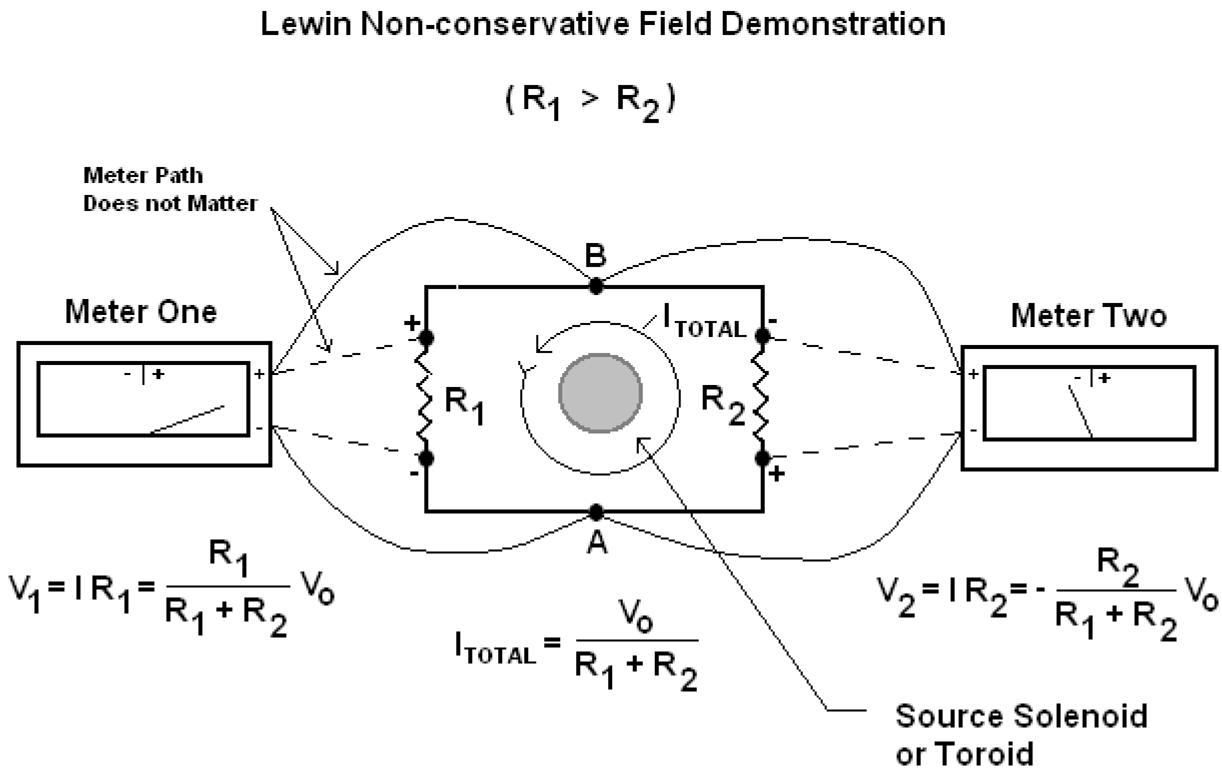

Fig 5. Lewin Non-Conservative Electric Field Demonstration

Here there are two meters both with the + lead connected to point B in the circuit and the negative lead connected to point A, yet meter one is reading a large positive value and meter two is reading a small negative value! Meters are assumed to prove no parallel resistance to resistors they are measuring.

But the “trick” giving a negative reading on one meter and a positive one on the other is easy to understand once we realize the current is circulating in the resistor loop. Convention says that leads with currents entering a passive element like a resistor are marked with a plus sign and a voltmeter meter connected with that polarity will show a positive reading. If we examine the above diagram carefully we notice that Meter One is connected to give a positive reading while Meter Two has the positive lead connected to the minus side of the resistor and the minus lead connected to the positive indicator. Hence meter two shows a negative reading.

But apart from polarity, there is the fact that two identical meters connected to the same points give different values of voltage. This is outside one's experience with common electric circuits and hence is confounding. Circuit theory requires conservative fields so that potential differences are always uniquely defined between two points.

The “trick” however is explained by remembering our investigations above where it becomes understood that the wires leading from the ends of the resistors to A or B are not simply wires, but are sources of voltage. And indeed even the resistors themselves are voltage sources due to the induction fields from the source solenoid. Thus, as we saw before, the part of the loop from A or B to the end of a resistor, is in essence a voltage source. Furthermore, the test lead going from the meter to A or B is also in the induction fields and therefore ALSO a voltage source. As we observed before these two voltage sources are back to back and thus cancel out. The net result is the meter reads the voltage across it's resistor as if it were connected directly to it. Since the load resistance of the voltmeters is assumed to be very high, they do not load the resistors they are connected to and the current going around the loop is the same in each resistor. Since the resistor resistance is different for each, then by Ohm's law the voltage across each will be different. The meter readings reflect this.

We know what to expect for the voltages across each resistor even though the resistors are receiving induction from the source coil, because we already calculated that. The final current is always equivalent to a voltage source equal to the emf produced by the source coil placed across the total resistance in the loop. In other words in this case:

$$I=\frac{V}{R}=I_{TOTAL}=\frac{V_o}{(R_1+R_2)}$$

This value is always true no matter what percentage of the current in the resistor is from the usual electrostatic potential and how much is from the induction electrokinetic field. $\mathbf{V_o}$ is the value of the emf induced for one turn about the magnetic field. Because of this equivalence the voltage across each resistor is simply calculated by Ohm' s law or:

$$\boldsymbol{V}_1=\frac{R_1}{R_1+R_2}V_o \quad \text{and} \quad V_2=-\frac{R_2}{(R_1+R_2)}V_o$$

An easy understanding of the “weird” measurements can be obtained from a quick examination of the “equivalent circuit” in Figure 6 where the induced voltages in the various wires have been replaced by equivalent batteries. The way the batteries in series “buck” each other explains how the meters are not actually measuring the voltage between A and B but are actually measuring the voltage across each resistor which are not equal in value.

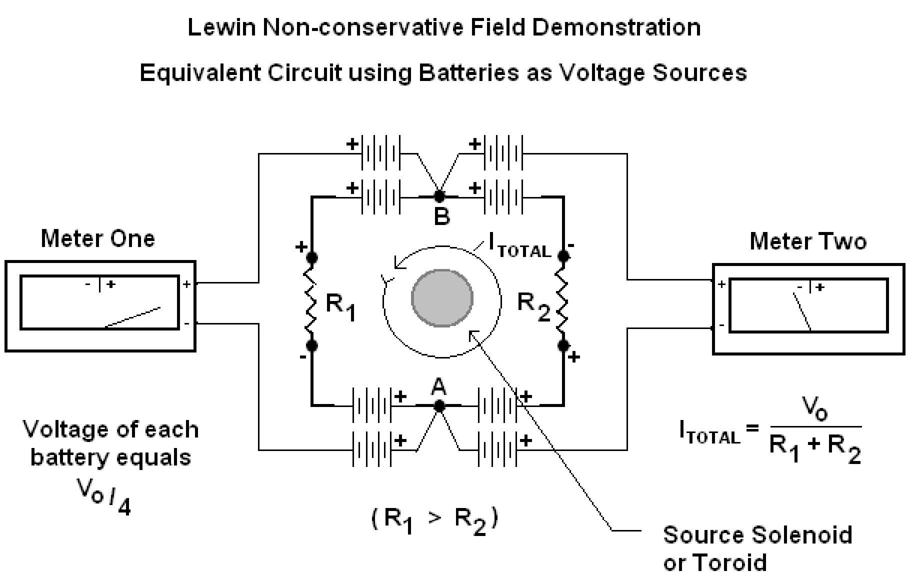

Fig 6. Equivalent circuit showing induction voltage sources in apparatus

While ***Fig. 6*** helps understand what is going on in the Lewin Demonstration, it is not precisely an equivalent circuit. One would not expect a conservative field circuit to cover all aspects of a non-conservative situation. For example the path independence effect is only implied and the equivalence we found no matter how much of the induction occurs withing the resistive material is glossed over in this model. Nevertheless if does provide a useful thinking tool for the study of induction and non-conservative electrokinetic fields.

As an interesting exercise, Romer presents a variant wiring in his paper as seen in ***Fig. 7.*** below. It is instructive to create the "equivalent circuit" with batteries for this arrangement showing that the both meters are reading the identically same negative $\mathbf{V_2}$. It is important to observe that the "equivalent circuit" with batteries for this layout is *not* the same as in the previous case. The equivalent circuit depends upon the path of the wires since the batteries are simply an approximation to the induced "source" created by the non-conservative electrokinetic electric fields.

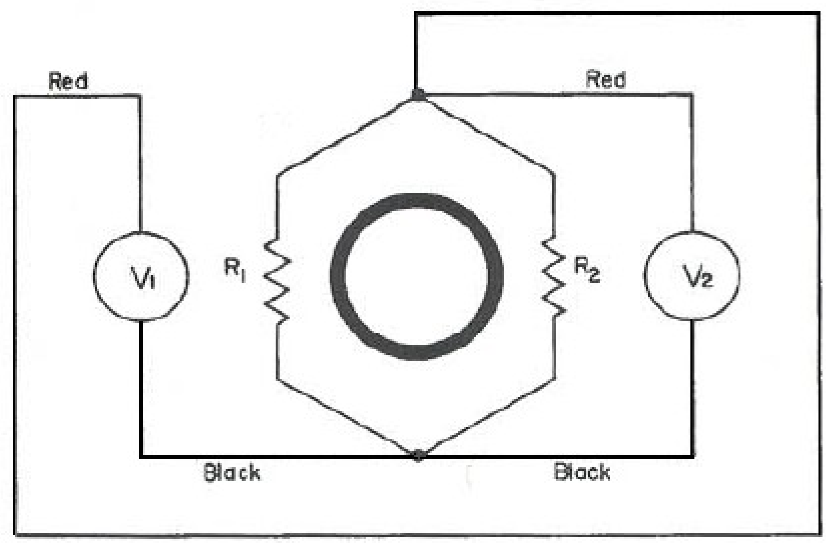

Fig. 6. Topology of this circuit is quite different from that of Fig. 1, and for this circuit, $V_1 = V_2$.

Fig. 7. Modification of Lewin Topology Taken from the Romer Paper.

This author has long suggested that after more than a century since its development, it is time to take a good hard look at Maxwell's theory in the light of discoveries and developments that have occurred since that time. Questioning authors such as Jefimenko, Romer, and Peng Kuan are promoting such a reevaluation.

The Lewin demonstration clearly achieved it's purpose of provoking thought and study among students and also serves to generate interest in a review of electromagnetics among the experienced as well. Romer notes in his paper: "*Of all the phenomena of physics, those associated with Faraday's law are among the most persistently fascinating and puzzling. How is it that* $\partial \boldsymbol{B}/\partial t$ *in one region demands the existence of curl* ***E*** *in that same region, and thus requires the existence of a non-vanshing* ***E*** *in other regions in which* ***B*** *and curl* ***E*** *both vanish?*"

How is it indeed?

Summary

This paper is completed with a list of the points made in the above discussion:

1. Faraday's law states that if there is a region of changing magnetic field, there is a related “electromagnetic force” that encircles it that can create currents. That force is obviously an electric field applying a “force” to charges in the region. This electric field is non-conservative.

2. Non-conservative electric fields such as those produced near examples of Faraday's law do not behave the same as the usual electrostatic electric fields of circuit theory. This can lead to apparent “impossible” circumstances such as two volt meters connected to the same two points producing entirely different readings.

3. The non-conservative electric field produced near examples of Faraday's law has been termed an *electrokinetic field* by Jefimenko due to it's properties that differ from an ordinary electrostatic field.

4. There has been promoted a doctrine of “one electric field” in electromagnetics for some time where all electric fields are treated as interchangeable and therefore can be blindly exchanged in calculations. However, it is observed that while all electric fields are force fields, they may have widely different properties. For example a conservative electrostatic electric field is conservative and irrotational while a non-conservative electokinetic electric field is solenoidal. A “*Lorentz electric field*” can result from the motion of charge in a magnetic field.

5. Jefimenko has shown that a Lorentz electric field falls naturally out of the mathematics of an electrokinetic field if currents are placed in motion and thus a Lorentz electric field is not needed to be calculated separately it is only necessary to include motion.

6. Since any general field can be shown to be able to be equal to the sum of two fields one irrotational and one solenoidal, it is clear that division into electrostatic and electrokinetic fields is natural for electric fields even though some authors have made such a division while others claim it was unnecessary.

7. The source of an electrostatic field, $\mathbf{E_S}$ is charge which is stationary. This field obeys an inverse square law and is distorted by the presence of other charges.

8. The source of an electrokinetic field is any current changing in time. This field does not obey an inverse square law and is not distorted by the presence of other charges. And electrokinetic field is uniform about a changing current element and is parallel or ant-parallel to the current. It exists everywhere in space that it has reached at the speed of light, even inside of perfect conductors.

9. The time rate of change of current produces multiple fields in space about itself. These include a magnetic field, an electrokinetic $\mathbf{E_K}$ field and a magnetic vector potential, **A**. Because these three fields all have the same source they are related to each other, however, they do *not* “cause” each other as they are all delayed which is termed “retarded”, from their source current. The widely held belief that a changing magnetic flux “causes” an emf is in error.

10. Electric fields of whatever type are force fields where the force, **F,** on a “test charge” q, is given by **F =** q**E**, where **F** and **E** are vectors in the same direction and **E** is an electric field of the Electrostatic, Electrokinetic or even Lorentz (v x B) type.

11. Since work is force times distance, it is of interest to compute a line integral of E ● dl which can be defined as a “voltage difference” between two end points. This voltage may or may not depend upon the path taken.

12. If a circular hoop of some good conductor such as heavy copper wire is placed about a changing magnetic field such as that in the center of a toroid or long solenoid coil and a thin cut is made in the hoop such that no current can flow Peng Kuan notes that since there are no currents there can be no motion of charges in the wire and thus the total electric field inside the wire must be zero.

13. What happens in the split loop is that the electrokinetic field applies a force to free charges in the wire pushing them toward one end. The other end of the wire experiences a loss of charge leading to equal and opposite charges across the gap. This movement of charges creates an electrostatic $\mathbf{E_S}$ field that exactly cancels the electrokinetic electric field $\mathbf{E_K}$ and results in no electric field in the wire.

14. A line integral of zero yields zero voltage around the loop, yet from experience we know that transformers actually work and produce voltages leading to a paradox!

15. How the electrokinetic electric field produced by a changing current behaves near a source, where the magnetic field is restricted in area (such as in the case of a toroid or long solenoid), can be learned from an examination of Faraday's law. It is found that line integrals of non-conservative fields in this case are “quasi” or “pseudo” conservative. In other words any closed path not enclosing any flux has a line integral identically zero, for either electrostatic or electrokinetic fields and the choice of patch does not matter so long as no flux is enclosed within it.

16. If the path circles the flux then the value of the line integral which is to say “voltage” is given by value produced by the changing current which we will term $\mathbf{V_o}$ and if the path circles the confined flux n times then the voltage is found to equal $\mathbf{nV_o}$ so that indeed path choice still does matter.

17. Circuit theory was invented to eliminate the complex calculations involving Maxwell's equations and replace them with simple algebra. Several approximations are needed to achieve this simplification. These include that all line integrals be conservative or quasi-conservative which is to say path independent, that there is no radiation or currents lost from any part of a circuit and that components are considered “ideal” such as wires having zero resistance or all inductance of actual capacitors being ignored.

18. Path independence in circuit theory means that a voltage difference reading taken with a voltmeter at any two points in a circuit will always be the same no matter how the test leads are brought to those two points. In other words the voltage at any point is defined and single valued in circuit theory.

19. A closer look at the split hoop situation shows that two voltmeters connected to the same to points A, B (across the gap) can indeed read differently. One reads zero the other reads $\mathbf{V_o}$**,** but careful examination shows that the problem is that voltages are being induced into the test leads that cancel the potential at the gap leading to a reading of zero.

20. Faraday's law provides a redundant explanation where it is observed that circuit of one meter reading $\mathbf{V_o}$ circles the toroid flux while the circuit of the meter reading zero encloses no flux. The anomaly is only apparent since everyone is used to normal circuit situations where there is no induction observed in test leads to meters or oscilloscopes.

21. Given that a wire in an inductive field produces charge shifts and an electrostatic field at it's ends, the next question that arises is what if part of the inductive loop is made of resistive material? We note that in this case field in the wire is essentially zero, but there are both the inductive electrokinetic field and the electostatic conservative field from the charge at the wire ends inside the resistive material.

22. Because there are two electric fields inside the resistor, there are two currents flowing. The ratio of these currents was found to be related to the ratio of the length of the resistive material to the length of the wire.

23. But it was found that the SUM of these two currents always produces a relationship according to Ohm's law where $\mathbf{V_o}$ which is the emf produced around the loop by Faraday's law is the voltage source and the current in the divided loop is given by $\mathbf{V_o / R}$ as if there were no induction into the resistive material at all, but just a normal voltage source into a typical passive resistor.

24. These derivations are combined to explain the apparently anomalous results of the Lewin demonstration where two meters connected to the same two points, A and B, read different polarities as well as different voltages. The secret, of course, is that there is induction of voltage not only into the loop with the resistors, but also into the meter leads.

25. Therefore, an “equivalent” circuit can be drawn with induction replaced by batteries as voltage sources that shows why it is that the two meters read differently.

26. And finally Romer changes the topology of the wiring which leads to vastly different readings. In this case both meters read the same and show the negative voltage across the resistor $\mathbf{R_2}$**.** The equivalent circuit for this arrangement is different from the former wiring showing that the common circuit is still not truly “equivalent” to the inductive setup.

27. In summary we note the lack of concern for inductive fields in electromagnetics heretofore, and the progress that could be made by a closer examination of the phenomena.

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As this is an open-source paper, as usual all comments, criticisms or corrections are welcomed and encouraged.